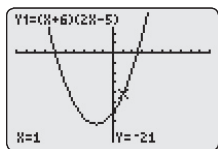


# Answers

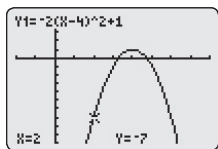
## CHAPTER 1

### Prerequisite Skills, pages 2–3

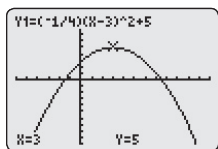
1. a) 7 b)  $-5$  c) 11 d) 5 e)  $8x + 7$  f)  $-12x + 7$   
 2. a) 1 b) 10 c) 6 d) 0 e)  $24x^2 - 18x + 3$  f)  $18x^2 - 9x + 1$   
 3. a)  $m = 3, b = 2$  b)  $m = -\frac{1}{2}, b = \frac{3}{2}$  c)  $m = 5, b = 7$   
 d)  $m = -5, b = -11$  e)  $m = -\frac{1}{2}, b = 1$   
 4. a)  $y = 3x + 5$  b)  $y = 4x + 4$  c)  $y = -4x + 31$   
 d)  $y = -7x + 12$   
 5. a) linear b) neither c) quadratic  
 6. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 1\}\}$  b)  $\{x \in \mathbb{R}, x \neq -5\}, \{y \in \mathbb{R}, y \neq 0\}$   
 c)  $\{x \in \mathbb{R}, x \leq \frac{1}{2}\}, \{y \in \mathbb{R}, y \geq 0\}$   
 7. Answers may vary. Sample answers:  
 a)  $y = -3(x + 1)(x - 1)$  b)  $y = -2x^2 - 3x + 3$   
 c)  $y = 4\left(x + \frac{1}{2}\right)(x - 2)$   
 8. a)  $x$ -intercepts  $-6, \frac{5}{2}$ ; vertex  $\left(-\frac{7}{4}, -\frac{289}{8}\right)$ ; opens up;  
 $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq -\frac{289}{8}\}$



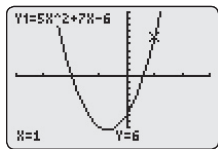
- b)  $x$ -intercepts approximately 3.29, 4.71; vertex (4, 1); opens down;  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq 1\}$



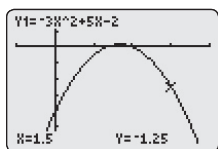
- c)  $x$ -intercepts approximately  $-1.47, 7.47$ ; vertex (3, 5); opens down;  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq 5\}$



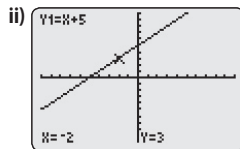
- d)  $x$ -intercepts  $-2, \frac{3}{5}$ ; vertex  $\left(-\frac{7}{10}, -\frac{169}{20}\right)$ ; opens up;  
 $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq -\frac{169}{20}\}$



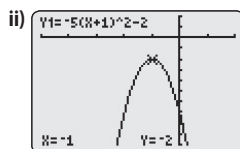
- e)  $x$ -intercepts  $1, \frac{2}{3}$ ; vertex  $\left(\frac{5}{6}, \frac{1}{12}\right)$ ; opens down;  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq \frac{1}{12}\}$



9. a) vertical stretch and a reflection in the  $x$ -axis b) vertical compression c) horizontal compression d) horizontal stretch and a reflection in the  $y$ -axis e) reflection in the  $y$ -axis  
 10. a) i)  $f(x) = x + 5$

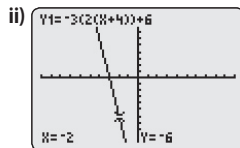


- iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$   
 b) i)  $f(x) = -5(x + 1)^2 - 2$



- iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq -2\}$

- c) i)  $f(x) = -3[2(x + 4)] + 6$



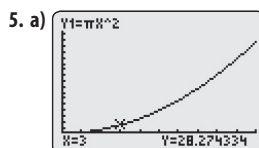
- iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

11. a) i) vertical stretch by a factor of 2, reflection in the  $x$ -axis, translation 3 units left, translation 1 unit up  
 ii)  $y = -2(x + 3) + 1$  b) i) vertical compression by a factor of  $\frac{1}{3}$ , translation 2 units down ii)  $y = \frac{1}{3}x^2 - 2$   
 12. vertical stretch by a factor of 3, horizontal stretch by a factor of 2, reflection in the  $y$ -axis, translation 1 unit right, translation 2 units up

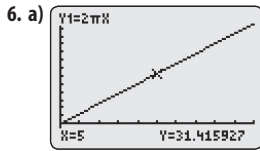
### 1.1 Power Functions, pages 11–14

1. a) No. b) Yes. c) Yes. d) Yes. e) No. f) No.  
 2. a) 4, 5 b) 1,  $-1$  c) 2, 8 d) 3,  $-\frac{1}{4}$  e) 0,  $-5$  f) 2, 1  
 3. a) i) even ii) negative iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq 0\}$  iv) line v) quadrant 3 to quadrant 4 b) i) odd ii) positive iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$  iv) point v) quadrant 3 to quadrant 1 c) i) odd ii) negative iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$  iv) point v) quadrant 2 to quadrant 4 d) i) even ii) negative iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq 0\}$  iv) line v) quadrant 3 to quadrant 4 e) i) odd ii) negative iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$  iv) point v) quadrant 2 to quadrant 4

End Behaviour	Function
Extends from quadrant 3 to quadrant 1	$y = 5x, y = 4x^5$
Extends from quadrant 2 to quadrant 4	$y = -x^3, y = -0.1x^{11}$
Extends from quadrant 2 to quadrant 1	$y = \frac{3}{7}x^2, y = 2x^4$
Extends from quadrant 3 to quadrant 4	$y = -x^6, y = -9x^{10}$



- b)  $\{r \in \mathbb{R}, 0 \leq r \leq 10\}; \{A \in \mathbb{R}, 0 \leq A \leq 100\pi\}$   
 c) Answers may vary. Sample answer: similarities—vertex (0, 0),  $x$ -intercept,  $y$ -intercept, end behaviour; differences—domain, range, shape

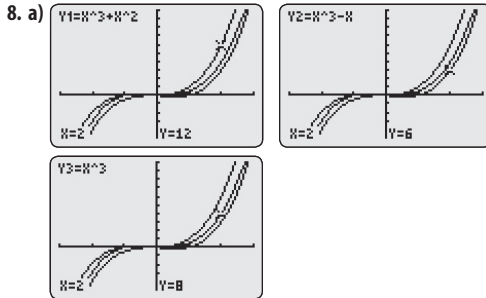


b)  $\{r \in \mathbb{R}, 0 \leq r \leq 10\}$ ,  $\{C \in \mathbb{R}, 0 \leq C \leq 20\pi\}$

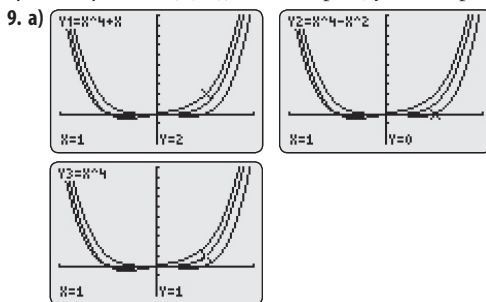
c) Answers may vary. Sample answer: similarities—end behaviour; differences—domain, range, shape

7. a) power (cubic) b) exponential c) periodic d) power (constant)

e) none of these f) none of these g) power (quadratic)

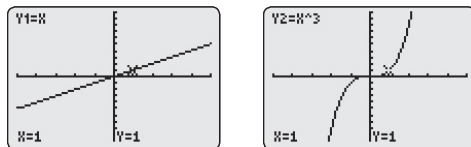


b)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ , quadrant 3 to quadrant 1, point symmetry about  $(0, 0)$ ;  $x$ -intercept 0,  $y$ -intercept 0

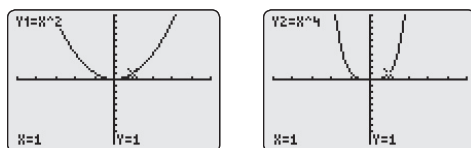


b)  $\{x \in \mathbb{R}\}$ , quadrant 2 to quadrant 1;  $x$ -intercept 0,  $y$ -intercept 0

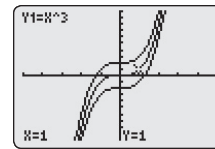
10. Answers may vary. Sample answer: similarities—extend from quadrant 1 to quadrant 3 (positive leading coefficient),  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ , point symmetry about  $(0, 0)$ ; differences—shape, extend from quadrant 2 to quadrant 4 (negative leading coefficient)



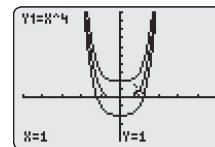
11. Answers may vary. Sample answer: similarities—extend from quadrant 2 to quadrant 1 (positive leading coefficient), domain, line symmetry; differences—shape, range, extend from quadrant 3 to quadrant 4 (negative leading coefficient)



12. a) Answers may vary. Sample answer: similarities—quadrant 3 to quadrant 1, domain, range, point symmetry, shape; difference—shifted vertically



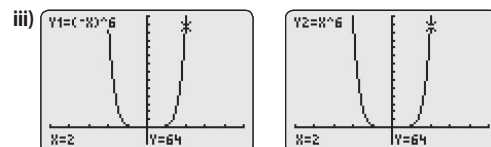
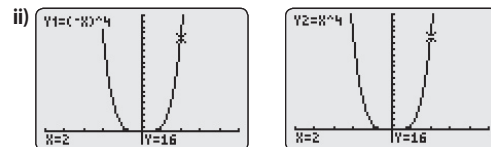
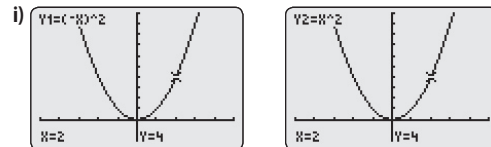
b) Answers may vary. Sample answer: similarities—quadrant 2 to quadrant 1, domain, line symmetry, shape; differences—range, shifted vertically



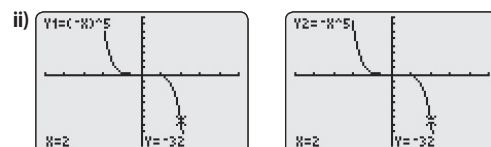
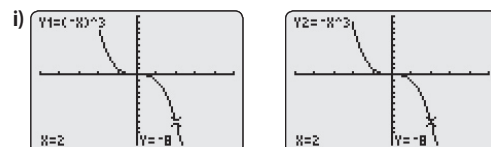
c)  $c$  is a vertical shift of  $x^n$ ,  $n \in \mathbb{N}$

13. Answers may vary. Sample answer: path of a river:  $y = x^3$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ ; cross-section of a valley:  $y = x^2$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$

14. a)  $y = (-x)^{2n}$  is the same graph as  $y = x^{2n}$ ,  $n$  is a non-negative integer,  $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$



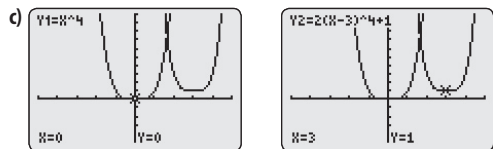
b)  $y = (-x)^{2n+1}$  has the same graph as  $y = -x^{2n+1}$ ,  $n$  is a non-negative integer,  $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$



c) Answers may vary. Sample answer:  $y = (-x)^{2n}$  has the same graph as  $y = x^{2n}$ ,  $n$  is a non-negative integer,  $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$ ;  $y = (-x)^{2n+1}$  has the same graph as  $y = -x^{2n+1}$ ,  $n$  is a non-negative integer,  $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$

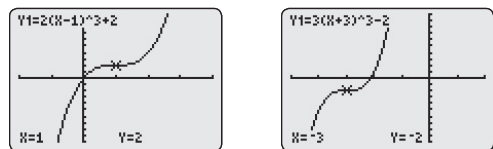
15. a) Answers may vary. Sample answer: For the graph of  $y = ax^n$ , if  $a > 0$ , vertical stretch by a factor of  $a$  if  $0 < a < 1$  vertical compression by a factor of  $a$ ; if  $1 < a < 0$ , vertical compression by a factor of  $a$  and a reflection in the  $x$ -axis; if  $a < -1$ , vertical stretch by a factor of  $a$  and a reflection in the  $x$ -axis

16. a) vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up b) vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up



17. a)  $a$  is a vertical stretch or compression;  $b$  is a shift left or right;  $k$  is a shift up or down

b) Answers may vary. Sample answers:



18. 124

19. (4, 6), (6, 9)

## 1.2 Characteristics of Polynomial Functions, pages 26–29

1. a) 4 b) 5 c) 4 d) 5 e) 3 f) 6

2. a)–d)

Graph	Sign of Leading Coefficient	End Behaviour (quadrants)	Symmetry	Local Maximum Points (#)	Local Minimum Points (#)
1a)	+	2 to 1		2	1
1b)	+	3 to 1		2	2
1c)	-	3 to 4		1	2
1d)	-	2 to 4		2	2
1e)	-	2 to 4	point	1	1
1f)	-	3 to 4	line	2	3

d) number of maximums and minimums is less than or equal to the degree of the function plus one; number of local maximums and local minimums is less than or equal to the degree of the function minus one

	i) End Behaviour (quadrants)	ii) Constant Finite Differences	iii) Value of Constant Finite Differences
a)	2 to 1	2nd	2
b)	2 to 4	3rd	-24
c)	3 to 4	4th	-168
d)	3 to 1	5th	72
e)	2 to 4	1st	-1
f)	3 to 4	6th	-720

4. a) 2, -4 b) 4, -2 c) 3, -2 d) 4, 1 e) 3, 6 f) 5,  $\frac{1}{2}$

5. a) odd b) even c) odd d) even

6.

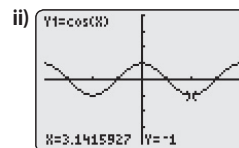
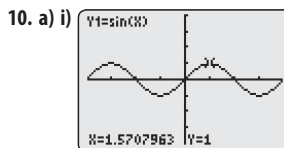
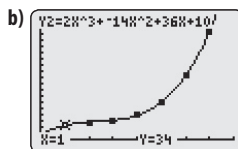
Graph	a) Least Degree	b) Sign of Leading Coefficient	c) End Behaviour (quadrants)	d) Symmetry
5a)	5	-	2 to 4	point
5b)	4	+	2 to 1	line
5c)	3	+	3 to 1	point
5d)	6	-	3 to 4	none

7. a) i) 3 ii) + iii) 1 b) i) 4 ii) - iii) -1

8. a) quartic b) fourth, 0.03 c) quadrant 2 to quadrant 1

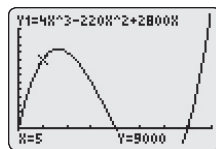
d)  $x \geq 0$  e) Answers may vary. Sample answer: They represent when the profit is equal to zero. f) \$1 039 500

9. a) i) cubic (degree 3) ii) 2



b) Answers may vary.

11. a)  $x \geq 0$ ,  $V(x) \geq 0$



b)  $V(x) = 4x(x - 35)(x - 20)$ ;  $x$ -intercepts 35, 20, 0 c) 24

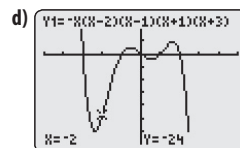
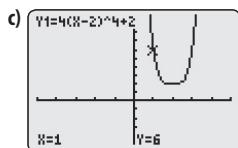
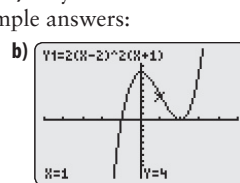
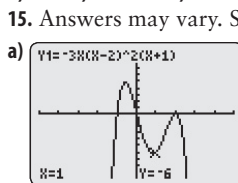
12. a) cubic b) third, -4.2 c) quadrant 2 to 4

d)  $\{d \in \mathbb{R}, d \geq 0\}$ ,  $\{r \in \mathbb{R}, r \geq 0\}$

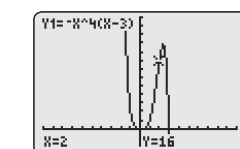
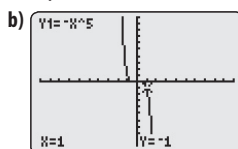
13. a) Answers may vary. Sample answer: quadrant 2 to quadrant 1,  $\{x \in \mathbb{R}\}$ ,  $\{P(t) \in \mathbb{R}, P(t) \geq 11\ 732\}$ , no  $x$ -intercepts

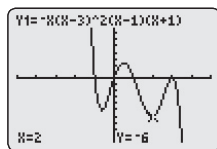
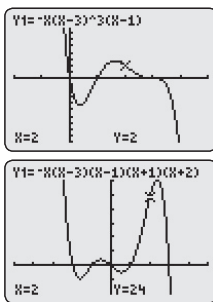
b) 144 c) 12 000 d) 69 000 e) 13 years

15. Answers may vary. Sample answers:

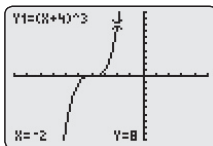
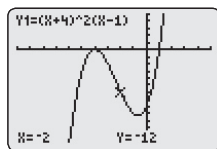
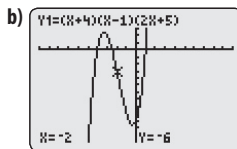


16. a) 1 to 5



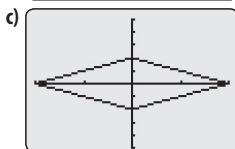
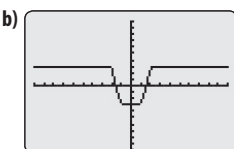
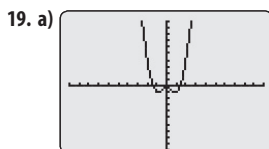


17. a) i) cubic ii) cubic iii) cubic



c) Answers may vary. Sample answer: The number of  $x$ -intercepts equals the number of roots of the equation.

18. a) i)  $S(r) = 6\pi r^2(r+1)$  ii)  $V(r) = 3\pi r^3$  b) Answers may vary. Sample answer:  $S(r)$  cubic, two  $x$ -intercepts,  $\{r \in \mathbb{R}\}$ ,  $\{S \in \mathbb{R}\}$ , quadrant 3 to quadrant 1;  $V(r)$  cubic, one  $x$ -intercept,  $\{r \in \mathbb{R}\}$ ,  $\{V \in \mathbb{R}\}$ , quadrant 3 to quadrant 1



### 1.3 Equations and Graphs of Polynomial Functions, pages 39–41

1. a) i) 3, + ii) quadrant 3 to quadrant 1 iii) 4,  $-3, \frac{1}{2}$

b) i) 4, - ii) quadrant 3 to quadrant 4 iii)  $-2, 2, 1, -1$

c) i) 5, + ii) quadrant 3 to quadrant 1 iii)  $-\frac{2}{3}, 4, -1, \frac{3}{2}$

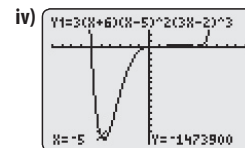
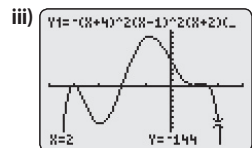
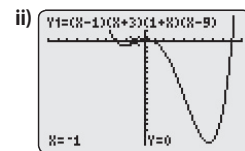
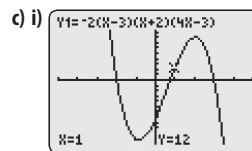
d) i) 6, - ii) quadrant 3 to quadrant 4 iii)  $-5, 5$

2. a) i)  $-4, -\frac{1}{2}$ , 1 ii) positive,  $-4 < x < -\frac{1}{2}, x > 1$ ; negative  $x < -4, -\frac{1}{2} < x < 1$  iii) no zeros of order 2 or 3

b) i)  $-1, 4$  ii) negative for all intervals iii) could have zeros of order 2 d) i)  $-3, 1$  ii) positive  $x < -3, x > 1$ ; negative  $-3 < x < 1$  iii) could have zeros of order 3 d) i)  $-5, 3$

ii) positive  $x < -5, -5 < x < 3$ ; negative  $x > 3$  iii) could have zeros of order 2 e) i)  $-2, 3$  ii) positive  $-2 < x < 3, x > 3$ ; negative  $x < -2$  iii) could have zeros of order 2 and 3

3. a) i)  $-2, 3, \frac{3}{4}$ , all order 1 ii)  $-3, -1, 1, 2, 3$ , all order 1 iii) order 2:  $-4, 1$ ; order 1:  $-2, \frac{3}{2}$  iv) order 3:  $\frac{2}{3}$ ; order 2:  $5$ ; order 1:  $-6$  b) graph in part ii) is even; others are neither



4. b) line, even; this function has line symmetry about the  $y$ -axis because it is an even function. a) c) d) neither, neither; there is no symmetry about the origin or about the  $y$ -axis because these functions are neither even nor odd.

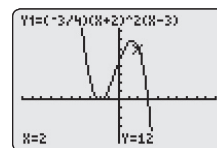
5. a) i) even ii) line b) i) odd ii) point c) i) neither ii) neither

d) i) neither ii) neither e) i) even ii) line

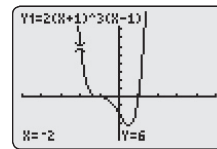
6. a)  $y = -2(x+3)(x+1)(x-2)$  b)  $y = -3(x+2)^2(x-1)^2$

c)  $y = 0.5(x+2)^3(x-1)^2$  d)  $y = (x+5)^3$

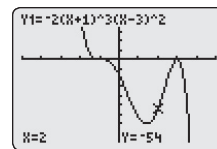
7. a)  $y = -\frac{3}{4}(x+2)^2(x-3)$ , neither



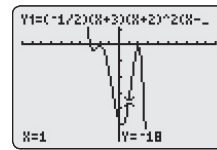
b)  $y = 2(x+1)^3(x-1)$ , neither



c)  $y = -2(x+1)^3(x-3)^2$ , neither

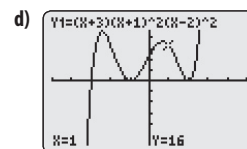
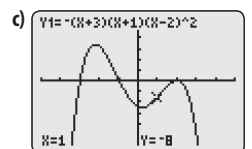
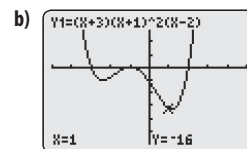
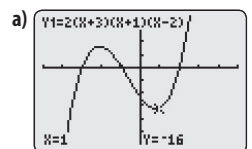


d)  $y = -\frac{1}{2}(x+3)(x+2)^2(x-2)^2$ , neither

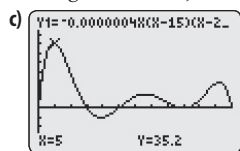


8. a) point b) line c) point d) point

9. Answers may vary. Sample answers:

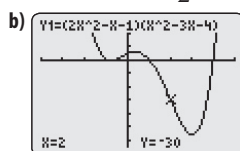


10. a) Answers may vary. b) Answers may vary. Sample answer: The equation provides information about the  $x$ -intercepts, the degree of the function, the sign of the leading coefficient, and the order of the zeros.

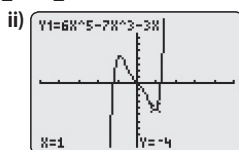
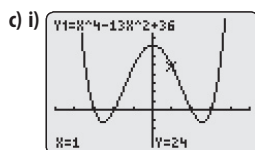


d) The maximum height is approximately 35.3 m above the platform. The minimum height is approximately 5.1 m below the platform.

11. a) 4, 1, -1,  $-\frac{1}{2}$

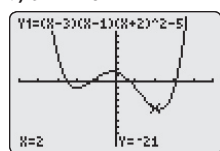


12. a) i) 3, 2, -2, -3 ii) 0,  $-\frac{\sqrt{6}}{2}$ ,  $\frac{\sqrt{6}}{2}$  b) i) even ii) odd

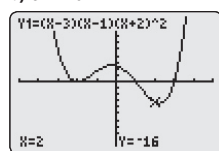


13. Answers may vary. Sample answers:

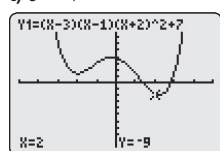
a)  $c = -5$



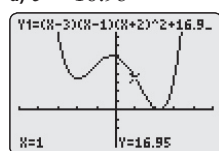
b)  $c = 0$



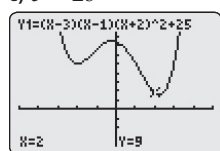
c)  $c = 7$



d)  $c \approx 16.95$



e)  $c = 25$



14. a) Answers may vary. Sample answer:

$$f(x) = (3x - 2)(3x + 2)(x - 5)(x + 5),$$

$$g(x) = 2(3x - 2)(3x + 2)(x - 5)(x + 5)$$

b)  $y = -3.2(3x - 2)(x - 5)$  c)  $y = 3.2(3x - 2)(x - 5)$

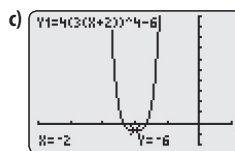
15. Answers may vary. Sample answer: shifts from the origin, and an odd function must go through the origin.

16. a) 0.35 b) 2.517

### 1.4 Transformations, pages 49–52

1. a)  $a = 4$  (vertical stretch by a factor of 4),  $k = 3$  (horizontal compression by a factor of  $\frac{1}{3}$ ),  $d = -2$  (translation 2 units left),  $c = -6$  (translation 6 units down)

$y = x^4$	$y = (3x)^4$	$y = 4(3x)^4$	$y = 4[3(x + 2)]^4 - 6$
(-2, 16)	(-2, 1296)	(-2, 5184)	(-2, -6)
(-1, 1)	(-1, 81)	(-1, 324)	(-1, 318)
(0, 0)	(0, 0)	(0, 0)	(0, 5178)
(1, 1)	(1, 81)	(1, 324)	(1, 26 238)
(2, 16)	(2, 1296)	(2, 5184)	(2, 82 938)



d)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq -6\}, (-2, -6), x = -2$

2. a) ii b) iv c) iii d) i

3. a) iii b) iv c) ii d) i

4. a)  $k = 3$  (horizontal compression by a factor of  $\frac{1}{3}$ ),  $c = -1$  (vertical translation 1 unit down),  $n = 3$

b)  $a = 0.4$  (vertical compression by a factor of 0.4),  $d = -2$  (horizontal translation of 2 units left),  $n = 2$  c)  $c = 5$  (vertical translation of 5 units up),  $n = 3$  d)  $a = \frac{3}{4}$  (vertical compression by a factor of  $\frac{3}{4}$ ),  $k = -1$  (reflection in the  $y$ -axis),  $d = 4$  (horizontal translation 4 units right),  $c = 1$  (vertical translation 1 unit up),  $n = 3$  e)  $a = 2$  (vertical stretch by a factor of 2),  $k = \frac{1}{3}$  (horizontal stretch by a factor of 3),  $c = -5$  (vertical translation 5 units down),  $n = 4$  f)  $a = 8$  (vertical stretch by a factor of 8),  $k = 2$  (horizontal compression by a factor of  $\frac{1}{2}$ ),  $c = 24$  (vertical translation 24 units up),  $n = 3$

5. a) ii b) iv c) i d) iii

6. a) 2 units left, 1 unit down,  $y = (x + 2)^2 - 1$

b) 4 units right, 5 units up,  $y = (x - 4)^3 + 5$

7. a)  $a = -3, k = \frac{1}{2}, d = -4, c = 1$

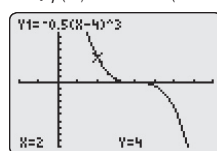
b)  $a$ : vertical stretch by a factor of 3 and a reflection in the  $x$ -axis;  $k$ : horizontal stretch by a factor of 2;  $d$ : 4 units left;  $c$ : 1 unit up c)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \leq 1\}, (-4, 1), x = -4$  d) vertical stretch, horizontal stretch, left, up; horizontal stretch, vertical stretch, up, left

8. a) vertical compression by a factor of 0.5 and a reflection in the  $x$ -axis, translation 4 units right;  $f(x) = -0.5(x - 4)^3$

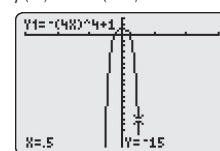
b) reflection in  $x$ -axis, horizontal compression by a factor of  $\frac{1}{4}$ , translation 1 unit up;  $f(x) = -(4x)^4 + 1$  c) vertical stretch by a factor of 2, horizontal stretch by a factor of 3, translation 5 units right, translation 2 units down;

$$f(x) = 2\left[\frac{1}{3}(x - 5)\right]^3 - 2$$

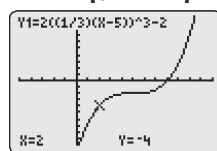
9. a)  $f(x) = -0.5(x - 4)^3$



$$f(x) = -(4x)^4 + 1$$



$$f(x) = 2\left[\frac{1}{3}(x - 5)\right]^3 - 2$$



b)  $f(x) = -0.5(x - 4)^3$ ,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$ ;  $f(x) = -(4x)^4 + 1$ ,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 1\}$ , vertex  $(0, 1)$ , axis of symmetry  $x = 0$ ;  
 $f(x) = 2\left[\frac{1}{3}(x - 5)\right]^3 - 2$ ,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

10. a) i)  $y = -x^4 + 2$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 2\}$ , vertex  $(0, 2)$ , axis of symmetry  $x = 0$  b) i)  $y = -(x - 5)^3$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

c) i)  $y = (x + 3)^4 - 5$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -5\}$ , vertex  $(-3, -5)$ , axis of symmetry  $x = -3$

11.  $y = x^3$ ,  $y = (x - 4)^3$ ,  $y = (x + 4)^3$ ,  $y = x^4 - 6$ ,  
 $y = (x - 4)^4 - 6$ ,  $y = (x + 4)^4 - 6$ ,  $y = -x^4 + 6$ ,  
 $y = -(x - 4)^4 + 6$ ,  $y = -(x + 4)^4 + 6$

12. a) i)  $f(x) = (x + 2)^4 + 3$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 3\}$ , vertex  $(-2, 3)$ , axis of symmetry  $x = -2$

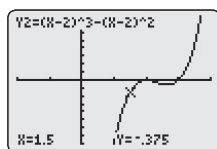
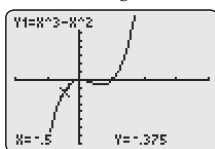
b) i)  $f(x) = \left[\frac{1}{5}(x + 12)\right]^5$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

c) i)  $f(x) = -3(x + 1)^4 - 6$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq -6\}$ , vertex  $(-1, -6)$ , axis of symmetry  $x = -1$

d) i)  $f(x) = -\left[\frac{1}{5}(x - 1)\right]^6 - 3$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq -3\}$ , vertex  $(1, -3)$ , axis of symmetry  $x = 1$

e) i)  $f(x) = -7\left[\frac{5}{4}(x + 1)\right]^4 + 9$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 9\}$ , vertex  $(-1, 9)$ , axis of symmetry  $x = -1$

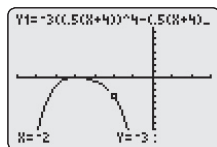
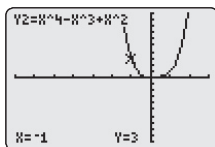
14. a) shift 2 right  
 b) i)  $f(x) = x^3 - x^2$  ii)  $f(x) = (x - 2)^3 - (x - 2)^2$



c) 0, 1; 3, 2

15. a) vertical stretch by a factor of 3, reflection in the  $x$ -axis, translation 4 units left

b) i)  $y = x^4 - x^3 + x^2$  ii)  $y = -3(0.5(x + 4))^4 - 0.5(x + 4)$  c) 0, -4



16. a)  $h(x) = 3(x + 1)(x + 6)(x - 1) - 5$

b)  $h(x) = -\frac{6}{5}(x + 1)(x + 6)(x - 1)$

17. \$3.12

18. a)  $x^4, x^8, x^{16}$  b)  $x^{2^n + 1}$

### 1.5 Slopes of Secants and Average Rate of Change, pages 62–64

1. e)

2. a) zero b) constant and positive c) constant and negative

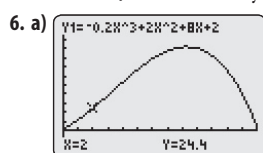
3. a) 0 b)  $\frac{7}{5}$  d)  $-\frac{2}{3}$

4. 3.89%/year

5. a) 6.45%/year

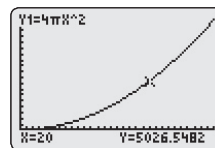
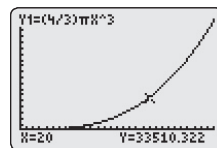
b) Answers may vary. c) 1999–2000: 11.1%/year; 2000–2001: 8.7%/year; 2001–2002: 2.8%/year; 2002–2003: 3.2%/year

d) greatest: between 1999 and 2000; least: between 2001 and 2002 e) Answers may vary.



b) positive  $0 \leq x < 8.28$ ; negative  $8.28 < x \leq 13$ ; zero:  $x = 8.28$  c) i) \$13/year ii) \$8.20/year iii) \$4.80/year iv) \$17.40/year d) Answers may vary.

7. a)  $V = \frac{4}{3}\pi r^3$ , cubic,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 0\}$



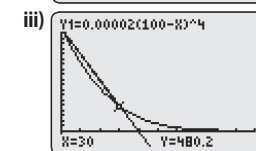
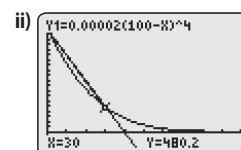
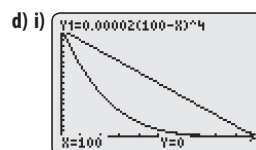
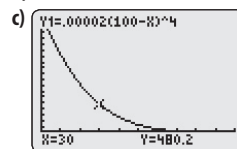
b) volume: i) 9529.50 ii) 6387.91; surface area: i) 691.15 ii) 565.49 c) 314.16 cm d) 570.07 e) Answers may vary.

8. Answers will vary.

9. a)  $y = -0.2x(x - 28)$  b) secant c) 2.4, 1.6, 0.4, 0, 0, 0, -0.4, -1.6, -2.4; calculated using two points on the curve d) steepness of the crossbeams e) have the same steepness in a different direction

10. a) i) -20 L/min ii) -50.66 L/min iii) -0.54 L/min

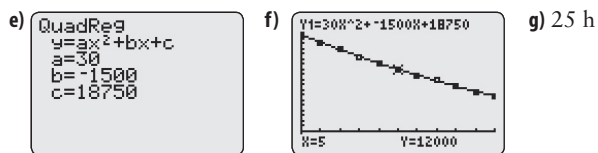
b) slows down



11. a) -1470, -1410, -1350, -1290, -1230, -1170, -1110, -1050, -990, -930 b) greatest: between 0 and 1; least: between 9 and 10 c) quadratic

Time (h)	Amount of Water (L)	Differences	
		First	Second
0	18 750		
1	17 280	-1470	
2	15 870	-1410	60
3	14 520	-1350	60
4	13 230	-1290	60
5	12 000	-1230	60
6	10 830	-1170	60
7	9 720	-1110	60
8	8 670	-1050	60
9	7 680	-990	60
10	6 750	-930	60

d) The first differences are the same as the average rates of change for the same interval.



12. a) velocity of the ball **b) i) 5.3 ii) 7.75 iii) 9.71 iv) 10.151 v) 10.195 vii) 10.200 c) instantaneous rate at 1 s**

13.  $b(p) = \frac{p^2 - 100}{2p}$

### 1.6 Slopes of Tangents and Instantaneous Rate of Change, pages 71–73

1. a) (5, 3) **b) (3, 7) c) -2 d) instantaneous rate of change at  $x = 5$**

2. a) A positive; B 0; C negative

b) A 4 m/s, C -6 m/s **c) velocity at 2 s and 7 s**

3. a)

Interval	$\Delta h$	$\Delta t$	$\frac{\Delta h}{\Delta t}$
$3 \leq t \leq 3.1$	-0.289	0.1	-2.89
$3 \leq t \leq 3.01$	-0.024 49	0.01	-2.449
$3 \leq t \leq 3.001$	-0.002 404 9	0.001	-2.4049

b) velocity is decreasing at a rate of approximately 2.4 m/s

4. 25 m/s

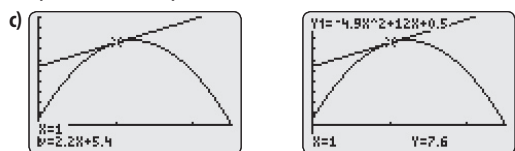
5. a) 3.9%/year **b) i) between 5.9% and 6.2% per year**

ii) between 1.3% and 2.2% per year **c) Answers may vary.**

6. a) 2.21 **b) i) between 0.3 and 0.84 ii) between 3.66 and 4.52**

iii) between 1.86 and 2.76 **c) Answers may vary.**

7. a) -15.2 m/s **b) 2.2 m/s**



d) average velocity between 1 s and 3 s (decelerating 15.2 m/s); velocity at 1 s is 2.2 m/s

8. a) Earth -44.10 m/s; Venus -40.05 m/s

b) Earth -29.44 m/s; Venus -26.7 m/s

c) a rock falls faster on Earth than on Venus

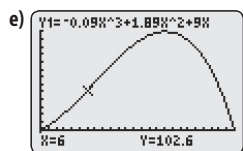
9. a) \$110.50/MP3 **b) \$118/MP3 c) Answers may vary.**

Sample answer: The cost of producing 200 MP3 players is more than the average cost of between 100 and 200 MP3 players. **d) No.**

10. a) \$327.25/MP3 **b) \$311/MP3 c) The average revenue (between 100 and 200) is more than the revenue for 200 MP3 players. d)  $P(x) = 250x - 0.000 475x^3$  e) \$216.75/MP3**

f) \$193/MP3 **g) The average profit between 100 and 200 MP3 players is more than the profit for 200 MP3 players.**

11. a) i) \$19 440 **ii) -\$10 800 b) Answers may vary. Sample answer: The profits are increasing between 2000 and 6000 basketballs and decreasing between 16 000 and 20 000 basketballs. c) i) \$21 150 ii) -\$10 440 d) losing money when making 18 000 basketballs**



12. a)  $0.5b^2 + 12b + 246$  **b) i) 272 ii) 302 iii) 318.5 c) Average rates of change between i) 8 and 10 years ii) 8 and 12 years iii) 8 and 13 years d) when  $b = 0$  e) 246**

13.  $\frac{1 \pm \sqrt{41}}{2}$

14.  $|m| = 4\sqrt{5}$

### Chapter 1 Review, pages 74–77

1. a) polynomial function, 4, 3 **b) polynomial function, 2, -1 e) polynomial function, 3, 5**

2. a) i) odd **ii) negative iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$  iv) quadrant 2 to quadrant 4 v) point **b) i) even ii) positive****

iii)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 0\}$  **iv) quadrant 2 to quadrant 1 v) line**

3.

End Behaviour	Function	Reasons
Extends from quadrant 3 to quadrant 1	c) $y = 4x^3$	positive leading coefficient and odd degree
Extends from quadrant 2 to quadrant 4	a) $y = -x^5$	negative leading coefficient and odd degree
Extends from quadrant 2 to quadrant 1	b) $y = \frac{2}{3}x^4$ , d) $y = 0.2x^6$	positive leading coefficient and even degree
Extends from quadrant 3 to quadrant 4	none	

4. a) iii **b) iv c) ii**

5. i) a) fourth **b) 12 c) none ii) a) fifth b) 120 c) point**

iii) a) sixth **b) -720 c) none**

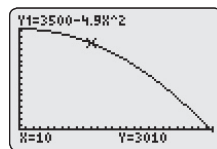
6. a) i) 1 **ii) 5 iii) 4 iv) 2 v) 3 vi) 3**

b) i) -5 **ii)  $-\frac{1}{2}$  iii)  $\frac{3}{2}$  iv) 9 v) 7 vi) -3**

7. a) i) 3 **ii) negative iii) -4 b) i) 5 ii) positive iii) 2**

8. a) quadratic **b) i) 2nd ii) -9.8 c) quadrant 3 to quadrant 4**

d)  $0 \leq t \leq 26.73, 0 \leq b \leq 3500$



e) when the parachutist lands

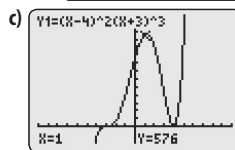
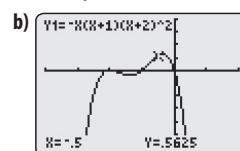
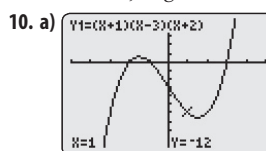
9. a) i) 3, positive **ii) 0, -2, 4 iii) positive  $-2 < x < 0, x > 4$ ; negative  $x < -2, 0 < x < 4$**

b) i) 4, negative **ii)  $-2, \frac{1}{2}, 3$  iii) positive  $-2 < x < \frac{1}{2}$ ;**

negative:  $x < -2, \frac{1}{2} < x < 3, x > 3$

c) i) 5, negative **ii) -3, -1, 3 iii) positive  $x < -3,$**

$-1 < x < 3$ ; negative  $-3 < x < -1, x > 3$



11. a) Answers may vary. Sample answer:

$y = (x + 3)(x + 1)(x - 2)^2, y = 2(x + 3)(x + 1)(x - 2)^2$

b)  $y = \frac{1}{2}(x + 3)(x + 1)(x - 2)^2$

12. a) point symmetry b) line c) neither

13. a)  $y = -(x + 2)(x - 4)^2$

b)  $y = -\frac{10}{27}(x + 2)^2(x - 0.5)(x - 4)$

14. a) i) vertical compression by a factor of  $\frac{1}{4}$ , reflection in the  $x$ -axis, translation 2 units down;  $y = -\frac{1}{4}x^3 - 2$

ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$  b) i) vertical stretch by a factor of 5, horizontal stretch by a factor of  $\frac{5}{2}$ , translation 3 units right, translation 1 unit up;  $y = 5\left[\frac{2}{5}(x - 3)\right]^4 + 1$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 1\}$ , vertex (3, 1), axis of symmetry  $x = 3$

15. a) i)  $y = \frac{3}{5}\left[\frac{1}{2}(x + 4)\right]^4 + 1$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 1\}$ , vertex (-4, 1), axis of symmetry  $x = -4$

b) i)  $y = -5[4(x + 2)]^3 + 7$  ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

16. a), c), d)

17. a) \$14, \$10 b) i) negative ii) positive iii) zero

c) i) -\$1.6/month ii) \$2/month iii) \$0/month

18. a) 5.7%/year b) between 6.7% and 6.9% in 2000; 4.6% in 2002 c) Answers may vary.

### Practice Test, pages 78–79

1. Answers may vary. Sample answers for false:

A. False;  $y = x^3 + x^2$

B. False;  $y = x^2$

C. True

D. False;  $y = x^4 + x$

2. Answers may vary. Sample answers for false:

A. False; degree  $n$  means constant  $n$ th differences

B. True

C. False; may have line symmetry

D. False; may have higher degree

3. Answers may vary. Sample answers for false:

A. False;  $y = \frac{1}{3}x^2$  does not equal  $y = \left(\frac{1}{3}x\right)^2$

B. False; stretches and compressions first

C. False; does not matter

D. False; reflection in the  $y$ -axis

E. True

4. a) i b) iii c) ii

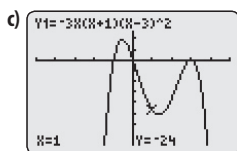
5. i) a) third b) -12 c) point ii) a) fifth b) 120 c) point

iii) a) sixth b) -720 c) line

6. a) Answers may vary. Sample answers:

$y = 2x(x + 1)(x - 3)^2$ ,  $y = -x(x + 1)(x - 3)^2$

b)  $y = -3x(x + 1)(x - 3)^2$



positive  $-1 < x < 0$ ; negative  $x < -1$ ,  $0 < x < 3$ ,  $x > 3$

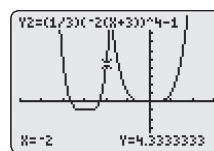
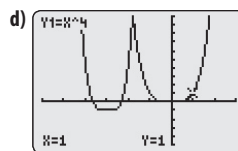
7.  $y = -x(x + 1)^2(x - 1)(x - 2)^3$

8. a)  $a = \frac{1}{3}$  (vertical compression by a factor of  $\frac{1}{3}$ ),  $k = -2$  (horizontal compression by a factor of  $\frac{1}{2}$ , reflection in the

$y$ -axis),  $d = -3$  (translation 3 units left),  $c = -1$  (translation 1 unit down)

b)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -1\}$ , vertex (-3, -1), axis of symmetry  $x = -3$

c) horizontal compression, vertical compression, reflection in the  $y$ -axis, 3 units left, 1 unit down horizontal compression, vertical compression, reflection in the  $y$ -axis, 1 unit down, 3 units left

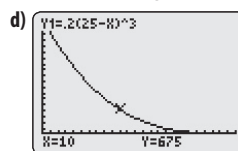


9.  $y = -2(x - 3)^3 - 5$

10. Answers may vary. Sample answer: a) distance versus time at a constant rate b) drop in temperature versus time at a constant rate c) acceleration d) no change in revenue over a period of time

11. 4.66%/year

12. a) 3125 L b) i) -245 L/min ii) -20 L/min c) Answers may vary. Sample answer: Average rate of change of volume over time is negative and increasing.



e) slopes of secants

13. a) 1 m/s b) 1.9 m/s c) speed is increasing

## CHAPTER 2

### Prerequisite Skills, pages 92–93

1. a) 124 R4 b) 161 R16 c) 147 R9 d) 358 R13

2. a) -22 b) -6 c) -51 d) -13.875 e)  $-\frac{169}{27}$

3. a)  $x^4 + x^3 - 7x^2 + 3x + 3$

b)  $2x^4 + 4x^3 - 15x^2 + x - 19$

c)  $3x^4 + 11x^3 - 7x^2 + 25x - 2$

d)  $x^2 - 2$  e)  $x^2 - 45$  f)  $x^2 - 2x - 2$

4. a)  $(x - 2)(x + 2)$  b)  $(5m - 7)(5m + 7)$

c)  $(4y - 3)(4y + 3)$  d)  $3(2c - 3)(2c + 3)$

e)  $2(x - 2)(x + 2)(x^2 + 4)$  f)  $3(n^2 - 2)(n^2 + 2)$

5. a)  $(x + 3)(x + 2)$  b)  $(x - 4)(x - 5)$

c)  $(b + 7)(b - 2)$  d)  $(2x + 3)(x - 5)$

e)  $(2x - 3)^2$  f)  $(2a - 1)(3a - 2)$

g)  $(3m - 4)^2$  h)  $(m - 3)(3m - 1)$

6. a)  $x = -3$  or  $x = 5$  b)  $x = -1$  or  $x = \frac{3}{4}$

c)  $x = -\frac{3}{2}$  or  $x = \frac{3}{2}$  d)  $x = \frac{1}{3}$  or  $x = 5$

e)  $x = -\frac{5}{2}$  or  $x = 1$  f)  $x = \frac{1}{7}$  or  $x = \frac{1}{3}$

7. a)  $x \neq -1.3$  or  $x \neq 0.1$

b)  $x \neq 0.7$  or  $x \neq 2.8$

c)  $x \neq -1.2$  or  $x \neq 0.7$

d)  $x \neq -1.3$  or  $x = 2.5$

8. a)  $y = -\frac{1}{3}(x + 4)(x - 1)$

b)  $y = -3x(x - 3)$  c)  $y = -4(x + 3)(x - 4)$

d)  $y = 2(x + 1)(x - 5)$  e)  $y = -3(2x + 1)(2x - 3)$

9. a) i) -4 and 1 ii) above the  $x$ -axis:  $x < -4$  and  $x > 1$ ; below the  $x$ -axis:  $-4 < x < 1$  b) i) -1, 1, and 2 ii) above the  $x$ -axis:  $-1 < x < 1$  and  $x > 2$ ; below the  $x$ -axis:  $x < -1$  and  $1 < x < 2$  c) i) -2, -1, 1, and 2 ii) above the  $x$ -axis:  $-2 < x < -1$  and  $1 < x < 2$ ; below the  $x$ -axis:  $x < -2$  and  $-1 < x < 1$  and  $x > 2$



## 2.1 The Remainder Theorem, pages 91–93

1. a)  $\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$  b)  $x \neq -1$   
 c)  $(x + 1)(x^2 + 2x - 4) + 9$   
 2. a)  $\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x - 4} = x^3 - 2x + 3 + \frac{4}{3x - 4}$   
 b)  $x \neq \frac{4}{3}$   
 c)  $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x - 4)(x^3 - 2x + 3) + 4$   
 3. a)  $\frac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \frac{30}{x + 2}$ ,  $x \neq -2$   
 b)  $\frac{6x^3 + x^2 - 14x - 6}{3x + 2} = 2x^2 - x - 4 + \frac{2}{3x + 2}$ ,  $x \neq -\frac{2}{3}$   
 c)  $\frac{10x^3 - 9x^2 - 8x + 11}{5x - 2} = 2x^2 - x - 2 + \frac{7}{5x - 2}$ ,  $x \neq \frac{2}{5}$   
 d)  $\frac{-4x^4 + 11x - 7}{x - 3} = 4x^3 - 12x^2 - 36x - 97 - \frac{298}{x - 3}$ ,  
 $x \neq 3$   
 e)  $\frac{6x^3 + x^2 + 7x + 3}{3x + 2} = 2x^2 - x + 3 - \frac{3}{3x + 2}$ ,  $x \neq -\frac{2}{3}$   
 f)  $\frac{8x^3 + 4x^2 - 31}{2x - 3} = 4x^2 + 8x + 12 + \frac{5}{2x - 3}$ ,  $x \neq \frac{3}{2}$   
 g)  $\frac{8x^3 + 6x^2 - 6x}{4x - 3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x - 3)}$ ,  $x \neq \frac{3}{4}$   
 4. a) 27 b) -9 c) -2  
 5.  $(x + 5)(x + 3)(2x + 1)$   
 6.  $(3x - 2)$  cm by  $(3x - 2)$  cm by  $(x + 4)$  cm  
 7. a) 16 b) 31 c) 36 d) 211 e) 4  
 8. a) 16 b) -13 c) -23  
 9. a) 9 b) 15 c) 41 d) -4  
 10. a)  $k = 3$  b) 123  
 11. a)  $c = 4$  b) 28  
 12.  $b = 11$   
 13.  $k = 3$   
 14. a) -c) 8  
 15. a) -b) 15  
 16. a)  $P\left(\frac{2}{3}\right) = 0$  b)  $(3x - 2)$  is a factor of  $6x^3 + 23x^2 - 6x - 8$ .  
 c)  $(3x - 2)(x + 4)(2x + 1)$   
 17. a)  $\pi(9x^2 + 24x + 16)$ ; this result represents the area of the base of the cylindrical container, i.e., the area of a circle.  
 b)  $\pi(3x + 4)^2(x + 3)$

Value of $x$	Radius (cm)	Height (cm)	Volume (cm <sup>3</sup> )
2	10	5	1 571
3	13	6	3 186
4	16	7	5 630
5	19	8	9 073
6	22	9	13 685
7	25	10	19 635
8	28	11	27 093

18. a)  $-5t^2 + 15t + 1 = (t - b)(-5t - 5b + 15) - 5b^2 + 15b + 1$  c)  $\frac{(-5t^2 + 15t + 1) - (-5b^2 + 15b + 1)}{t - b}$   
 d) Answers may vary. Sample answer: At  $t = b$ , there is a hole in the graph; the graph is discontinuous at  $t = b$ .  
 e) 1; at 3 s, the height of the javelin is 1 m.  
 19. a)  $\frac{12}{5}$  b) At 1.5 s, the shot-put is 2.4 m above the ground.  
 20.  $m = -\frac{11}{5}$ ,  $n = \frac{59}{5}$   
 21.  $a = -\frac{14}{3}$ ,  $b = -\frac{2}{3}$

22.  $k = -\frac{2}{3}$  or  $k = 4$

23. 3

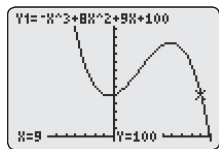
24.  $A \cong 8$

## 2.2 The Factor Theorem, pages 102–103

1. a)  $x - 4$  b)  $x + 3$  c)  $3x - 2$  d)  $4x + 1$   
 2. a) No. b) Yes. c) Yes.  
 3. a)  $(x - 2)(x + 1)(x + 4)$  b)  $(x - 3)(x + 1)(x + 6)$   
 c)  $(x - 4)(x - 2)(x + 3)$   
 4. a)  $(x - 3)(x + 1)(x + 3)$  b)  $(x - 4)(x - 1)(x + 4)$   
 c)  $(x - 6)(x + 6)(2x - 1)$  d)  $(x - 7)(x - 2)(x + 2)$   
 e)  $(x - 5)(x + 5)(3x + 2)$  f)  $x(x - 4)(x + 4)(2x + 3)$   
 5. a)  $(x - 3)(x + 2)(3x + 4)$  b)  $(x - 3)(x - 1)(2x - 1)$   
 c)  $(x - 3)(2x - 1)(3x + 5)$  d)  $(x - 1)(x + 1)(4x + 3)$   
 6. a)  $(x - 1)(x + 1)(x + 2)$  b)  $(x - 2)(x + 1)(x + 5)$   
 c)  $(x - 5)(x - 2)(x + 2)$  d)  $(x + 4)(x^2 + x - 1)$   
 e)  $(x - 5)(x - 2)(x + 3)$  f)  $(x - 3)(x + 2)(x - 1)(x - 2)$   
 g)  $(x - 4)(x - 2)(x + 1)(x + 3)$   
 7. a)  $(2x - 1)(2x + 1)^2$  b)  $(x - 1)(x + 2)(2x + 3)$   
 c)  $(x - 1)(x + 2)(5x - 2)$  d)  $(x - 1)(x + 1)(2x - 1)(3x + 2)$   
 e)  $(x - 2)(x + 2)(5x^2 + x - 2)$  f)  $(x - 3)(x + 4)(3x + 1)$   
 g)  $(x - 2)(2x - 1)(3x - 1)$   
 8.  $(x + 4)(2x - 1)(3x + 2)$   
 9.  $k = -3$   
 10.  $k = -1$   
 11. a)  $(x - 1)(x + 2)(2x + 3)$  b)  $(x + 1)(2x - 3)(2x + 1)$   
 c)  $(x - 1)(2x + 5)(3x - 2)$  d)  $(x - 2)(4x^2 + 3)$   
 e)  $(2x - 1)(x^2 + x + 1)$  f)  $(x - 4)(x - 1)(x + 2)(x + 3)$   
 12. a) i)  $(x - 1)(x^2 + x + 1)$  ii)  $(x - 2)(x^2 + 2x + 4)$   
 iii)  $(x - 3)(x^2 + 3x + 9)$  iv)  $(x - 4)(x^2 + 4x + 16)$   
 b)  $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$  c)  $(x - 5)(x^2 + 5x + 25)$   
 d) i)  $(2x - 1)(4x^2 + 2x + 1)$  ii)  $(5x^2 - 2)(25x^4 + 10x^2 + 4)$   
 iii)  $(4x^4 - 3)(16x^8 + 12x^4 + 9)$   
 iv)  $\left(\frac{2}{5}x - 4y^2\right)\left(\frac{4}{25}x^2 - \frac{8}{5}xy^2 + 16y^4\right)$   
 13. a) i)  $(x + 1)(x^2 - x + 1)$  ii)  $(x + 2)(x^2 - 2x + 4)$   
 iii)  $(x + 3)(x^2 - 3x + 9)$  iv)  $(x + 4)(x^2 - 4x + 16)$   
 b)  $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$  c)  $(x + 5)(x^2 - 5x + 25)$   
 d) i)  $(2x + 1)(4x^2 - 2x + 1)$  ii)  $(5x^2 + 2)(25x^4 - 10x^2 + 4)$   
 iii)  $(4x^4 + 3)(16x^8 - 12x^4 + 9)$   
 iv)  $\left(\frac{2}{5}x - 4y^2\right)\left(\frac{4}{25}x^2 - \frac{8}{5}xy^2 + 16y^4\right)$   
 14.  $(x^2 + x + 1)(x^2 - x + 1)$   
 15. a)  $(x - 3)(x + 3)(2x - 1)(2x + 1)$   
 b)  $(x - 4)(x + 4)(3x - 2)(3x + 2)$   
 17. a)  $(x - 2)(x - 1)(x + 1)(x + 2)(2x + 3)$   
 b)  $(x - 2)(x + 1)(x + 2)^2(2x - 1)(2x + 1)$   
 18.  $m = 0.7$ ,  $n = -0.9$   
 19. a)  $(x + 4)(4x + 3)(2x - 1)$   
 b)  $\frac{9}{10}(x - 3)(x + 1)(3x - 2)(2x + 3)$   
 20. a) i)  $(x - 1)(x + 1)(x^2 + 1)$  ii)  $(x - 2)(x + 2)(x^2 + 4)$   
 iii)  $(x - 1)(x^4 + x^3 + x^2 + x + 1)$   
 iv)  $(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$   
 b)  $x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-3}x^2 + a^{n-2}x + a^{n-1})$ , where  $n$  is a positive integer.  
 c)  $(x - 1)(x^5 + x^4 + x^3 + x^2 + x + 1)$   
 d) i)  $(x - 5)(x^2 + 25)$  ii)  $(x - 3)(x^4 + 3x^3 + 9x^2 + 27x + 81)$   
 21. Yes, but only if  $n$  is odd. Let  $n = 2k + 1$ .  
 Then,  $x^{2k+1} + a^{2k+1} = x^{2k} - x^{2k-1}a + x^{2k-2}a^2 - x^{2k-3}a^3 + \dots - xa^{2k-1} + a^{2k}$ . If  $n$  is even, then  $x^n + a^n$  is not factorable.  
 22.  $7x - 5$

### 2.3 Polynomial Equations, pages 110–112

1. a)  $x = 0$  or  $x = -2$  or  $x = 5$   
 b)  $x = 1$  or  $x = 4$  or  $x = -3$   
 c)  $x = -\frac{2}{3}$  or  $x = -9$  or  $x = 2$   
 d)  $x = 7$  or  $x = -\frac{2}{3}$  or  $x = -1$   
 e)  $x = 0.25$  or  $x = 1.5$  or  $x = -8$   
 f)  $x = 2.5$  or  $x = -2.5$  or  $x = 7$   
 g)  $x = 1.6$  or  $x = -3$  or  $x = 0.5$   
 2. a)  $x = -3$  or  $x = -1$  or  $x = 1$   
 b)  $x = -1$  or  $x = 3$  or  $x = 4$   
 c)  $x = -2$  or  $x = -1$  or  $x = 2$  or  $x = 3$   
 d)  $x = -5$  or  $x = -2$  or  $x = 1$   
 e)  $x = -3$  or  $x = -1$  or  $x = 0$  or  $x = 2$   
 3. a)  $x = 4$  b)  $x = 1$  or  $x = -1$  c)  $x = 4$  or  $x = -4$   
 d)  $x = -1$  or  $x = 1$  or  $x = 5$  or  $x = -5$   
 e)  $x = 1.5$  or  $x = -1.5$   
 f)  $x = 7$  or  $x = -7$  or  $x = -3$  or  $x = -4$   
 g)  $x = -3$  or  $x = 0.5$  or  $x = 5$  or  $x = -5$   
 4. a)  $-5, 0, 9$  b)  $-9, 0, 9$  c)  $-\frac{1}{2}, 0, \frac{4}{3}$   
 d)  $-2, -1, 2$  e)  $-2, 2$  f)  $-1, 0, 1, 2$  g)  $-5, -2, 2, 5$   
 5. Answers may vary. Sample answers:  
 a) False. If the graph of a quartic function has four  $x$ -intercepts, then the corresponding quartic equation has four real roots.  
 b) True. c) False. A polynomial equation of degree 3 has three or fewer real roots. d) False. If a polynomial equation is not factorable, the roots can be determined by graphing. e) True.  
 6. a)  $x = -2$  or  $x = 3$  b)  $x = 5$  or  $x = -2$  or  $x = 1$   
 c)  $x = 1$  or  $x = 3$  d)  $x = -2$  or  $x = 3$   
 e)  $x = -2$  or  $x = 2$  or  $x = 3$  f)  $x = -4$  or  $x = 1$  g)  $x = -1$   
 7. a)  $x = -2$  or  $x = -1$  or  $x = 1.5$  b)  $x = -0.5$  or  $x = 3$   
 c)  $x = -2$  or  $x = -\frac{2}{3}$  or  $x = \frac{2}{3}$   
 d)  $x = -2$  or  $x = 0.6$  or  $x = 3$  e)  $x = 0$  or  $x = 2$   
 f)  $x = -2$  or  $x = 0$  or  $x = 0.5$  or  $x = 2$   
 g)  $x = -3$  or  $x = -1$  or  $x = 2$  or  $x = 3.6$   
 8. a)  $x = -1$  or  $x = 2$  or  $x = 4$  b)  $x = 3$   
 c)  $x = 1$  d)  $x = -1$  or  $x = 2$  e) no real roots  
 9. a)  $x \doteq -2.2$  or  $x \doteq 0.5$  or  $x \doteq 1.7$   
 b)  $x \doteq -4.5$  or  $x \doteq -0.6$  or  $x \doteq 0.6$   
 c)  $x \doteq -1.2$  or  $x \doteq 1.2$  d)  $x \doteq -1.3$   
 e)  $x \doteq -1.4$  or  $x \doteq 1.9$  f)  $x = -1$  or  $x \doteq 0.4$  or  $x \doteq 1.4$   
 g) no real roots  
 10. 2 m by 2 m by 5 m  
 11. 13 m by 3 m by 3 m  
 12. Answers may vary. Sample answer: Yes, for example,  $x^3 + 2$ .  
 13. Answers may vary. Sample answer: No. If the radical part of the quadratic formula is negative, then two non-real roots occur.  
 14. 7 h  
 15. 0 m or 8 m or approximately 12.9 m from the end  
 16. a)  $\{x \in \mathbb{R}, 0 < x < 992\}$  b) 22 000



c)  $x = 3$  or  $x = 8$ ; If the selling price is \$3 per bottle or \$8 per bottle, then 17 200 bottles of sunscreen will be sold per month.

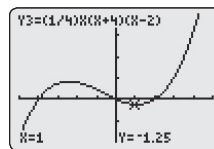
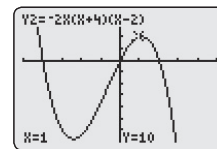
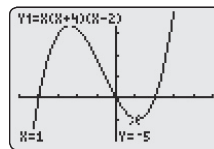
17. a)  $x = 3$  b)  $x \doteq -0.6$  or  $x \doteq 0.1$  or  $x \doteq 3.9$  or  $x \doteq 4.6$

18. a)  $k = 3$  b)  $x \doteq -1.3$  or  $x \doteq 0.8$   
 19. 24 cm by 20 cm by 4 cm, or 20 cm by 16 cm by 6 cm  
 20. a)  $x = 3$  or  $x = \frac{-3 + 3i\sqrt{3}}{2}$  or  $x = \frac{-3 - 3i\sqrt{3}}{2}$   
 b)  $x^3 - 2x^2 - 14x + 40$   
 21. 9 cm by 8 cm by 7 cm or 12 cm by 10 cm by 8 cm  
 22.  $x^3 - 4x^2 - \frac{69}{4}x + \frac{63}{2} = 0$   
 23.  $15^\circ$   
 24. 20

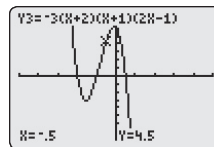
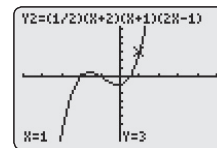
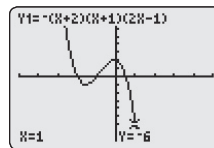
### 2.4 Families of Polynomial Functions, pages 119–122

1. a)  $y = k(x + 7)(x + 3)$ ,  $k \in \mathbb{R}$ ,  $k \neq 0$   
 b) Answers may vary. Sample answer:  
 $y = 2(x + 7)(x + 3)$ ,  $y = -3(x + 7)(x + 3)$   
 c)  $y = \frac{2}{5}(x + 7)(x + 3)$   
 2. C (has different zeros)  
 3. A, B and D (same zeros)  
 4. A, C, E (zeros are  $-3, -2, 1$ ); B, D, F (zeros are  $-1, 2, 3$ )  
 5. a)  $y = k(x + 5)(x - 2)(x - 3)$   
 b)  $y = k(x - 1)(x - 6)(x + 3)$   
 c)  $y = k(x + 4)(x + 1)(x - 9)$   
 d)  $y = kx(x + 7)(x - 2)(x - 5)$   
 6. a) A:  $y = (x + 2)(x - 1)(x - 3)$ ;  
 B:  $y = -\frac{1}{2}(x + 2)(x - 1)(x - 3)$ ;  
 C:  $y = -\frac{1}{2}(x + 2)(x - 2)(x - 3)$ ;  
 D:  $y = 2(x + 2)(x - 1)(x - 3)$   
 7. a)  $y = kx(x + 4)(x - 2)$  b) Answers may vary. Sample answer:  $y = x(x + 4)(x - 2)$ ,  $y = -2x(x + 4)(x - 2)$   
 c)  $y = \frac{1}{4}x(x + 4)(x - 2)$  d) Answers may vary.

Sample answers:



8. a)  $y = k(x + 2)(x + 1)(2x - 1)$   
 b) Answers may vary. Sample answer:  
 $y = -(x + 2)(x + 1)(2x - 1)$ ,  $y = \frac{1}{2}(x + 2)(x + 1)(2x - 1)$   
 c)  $y = -3(x + 2)(x + 1)(2x - 1)$   
 d) Answers may vary. Sample answer:



9. a)  $y = k(x + 4)(x + 1)(x - 2)(x - 3)$

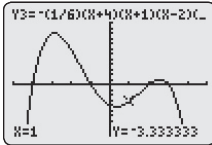
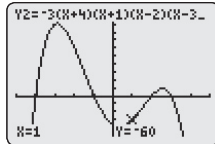
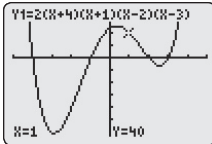
b) Answers may vary. Sample answer:

$$y = 2(x + 4)(x + 1)(x - 2)(x - 3),$$

$$y = -3(x + 4)(x + 1)(x - 2)(x - 3)$$

c)  $y = -\frac{1}{6}(x + 4)(x + 1)(x - 2)(x - 3)$

d) Answers may vary. Sample answer:



10. a)  $y = k(2x + 5)(x + 1)(2x - 7)(x - 3)$

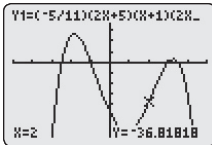
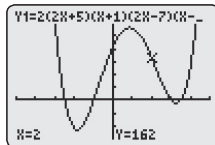
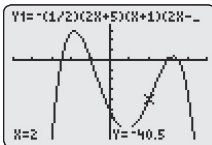
b) Answers may vary. Sample answer:

$$y = -\frac{1}{2}(2x + 5)(x + 1)(2x - 7)(x - 3),$$

$$y = 2(2x + 5)(x + 1)(2x - 7)(x - 3)$$

c)  $y = -\frac{5}{11}(2x + 5)(x + 1)(2x - 7)(x - 3)$

d) Answers may vary. Sample answer:



11. a)  $y = k(2x^3 - 3x^2 - 4x - 1)$

b)  $y = \frac{35}{14}(2x^3 - 3x^2 - 4x - 1)$

12. a)  $y = k(x^4 + 2x^3 - 26x^2 - 6x + 117)$

b)  $y = -\frac{1}{4}(x^4 + 2x^3 - 26x^2 - 6x + 117)$

13. a)  $y = k(x^4 - 2x^3 - 10x^2 + 20x - 8)$

b)  $y = 4(x^4 - 2x^3 - 10x^2 + 20x - 8)$

14.  $y = -2(x + 2)(x - 1)(x - 3)$

15.  $y = (x + 3)^2(x - 1)(2x - 3)$

16.  $y = -2x(2x + 7)(x + 2)(x - 1)$

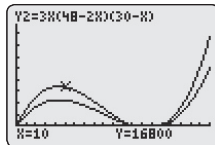
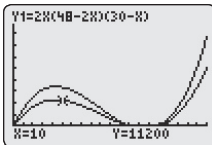
17. Set A: no; Set B: yes

18. a)  $V = x(48 - 2x)(30 - x)$  b) 44.31 cm by 28.16 cm

by 1.84 cm or 18.6 cm by 11.4 cm by 10.8 cm

c) volume doubles; volume triples; family of functions

with zeros 24, 30, 0 d) Answers may vary. Sample answer:

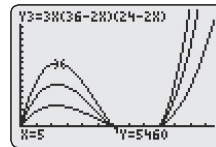
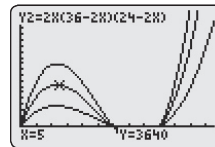
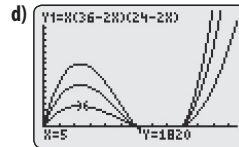


19.  $y = kx(x - 30)(x - 60)(x - 90)(x - 120)(x - 150)$

20. a)  $V = x(36 - 2x)(24 - 2x)$

b) i)  $V = 2x(36 - 2x)(24 - 2x)$  ii)  $V = 3x(36 - 2x)(24 - 2x)$

c) family of functions with the same zeros



e) approximately 27.16 cm by 15.16 cm by 4.42 cm or 26 cm by 14 cm by 5 cm

22. a) Answers may vary. Sample answer:

$$y = k(3x - 2)(x - 5)(x + 3)(x + 2)$$

b) 4 c) Answers may vary. Sample answer:

$$y = -\frac{8}{5}(3x - 2)(x - 5)(x + 3)(x + 2)$$

d)  $y = \frac{8}{5}(3x - 2)(x - 5)(x + 3)(x + 2)$

23. Answers may vary.

24. 24 cm

25.  $g(x^2 - 1) = x^4 - x^2 - 3$

## 2.5 Solving Inequalities Using Technology, pages 129–131

1. a)  $-7 < x \leq -1$  b)  $-2 < x \leq 6$  c)  $x < -3, x \geq 4$

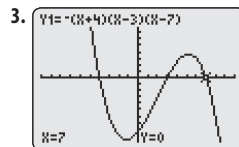
d)  $x \leq -1, x \geq 1$

2. a)  $x < -1, -1 < x < 5, x > 5$

b)  $x < -7, -7 < x < 0, 0 < x < 2, x > 2$

c)  $x < -6, -6 < x < 0, 0 < x < 1, x > 1$

d)  $x < -4, -4 < x < -2, -2 < x < \frac{2}{5}, \frac{2}{5} < x < 4.3, x > 4.3$



4. a)  $f(x) > 0$  when  $x < -2$  or  $1 < x < 6$

b)  $f(x) < 0$  when  $-3.6 < x < 0$  or  $x > 4.7$

5. a) i)  $-6, 3$  ii)  $-6 < x < 3$  iii)  $x < -6, x > 3$

b) i)  $-2, 5$  ii)  $x < -2, x > 5$  iii)  $-2 < x < 5$

c) i)  $-4, 3$  ii)  $-4 < x < 3, x > 5$  iii)  $x < -4, 3 < x < 5$

d) i)  $-4, 1$  ii)  $x < -4$  iii)  $-4 < x < 1, x > 1$

6. a)  $-3 < x < 4$  b)  $-5 \leq x \leq -3$

c)  $1 < x < 2, x > 3$  d)  $-4 \leq x \leq -3, x \geq -1$

e)  $x < -3, 2 < x < 3$  f)  $x \leq -4, -1 \leq x \leq 4$

7. a)  $x \leq -4$  or  $x \geq 0.5$  b)  $-0.5 < x < 3$

c)  $x \leq -4$  or  $-2 \leq x \leq 1$  d)  $-5 < x < -1$  or  $x > 4$

e)  $x < -5$  or  $-2 < x < 7$  f)  $x \leq 7$

8. a)  $-4.65 < x < 0.65$  b)  $-2.43 < x < 1.10$

c)  $x \leq -2.17$  or  $-0.31 \leq x \leq 1.48$

d)  $-2.12 \leq x \leq -0.43$  or  $x \geq 0.55$

e)  $x < -1.93$  or  $-0.48 < x < 1.08$

f)  $-1.34 \leq x \leq 1.25$

9. a) approximately  $x > -0.67$

b)  $x \leq -4$  or  $-2 \leq x \leq 6$  c)  $x \leq -4$  or  $-\frac{1}{3} \leq x \leq 3$

d)  $x < -\frac{5}{3}$  or  $-1 < x < 2$  e)  $x < -2$  or  $-\frac{1}{2} < x < 3$

f)  $x < -3$  or  $-1 < x < -\frac{1}{2}$  or  $x > 4$

10. approximately  $0.50 < t < 6.03$ , or between about 0.5 s and 6.03 s.

11. a) approximately  $2.73 < t < 5.51$ , or between later in the second week and halfway through the fifth week

b) There are no tent caterpillars left.

12. a) between 0 and approximately 4.47 years

b) after approximately 4.94 years

13.–14. Answers may vary.

15. Answers may vary. Sample answers:

a)  $(3x + 2)(5x - 4)(2x - 7) > 0$ ,  
 $-30x^3 - 109x^2 - 2x - 56 < 0$

b)  $x^3 - 2x^2 - 10x + 8 > 0$ ,  $-x^3 + 2x^2 + 10x - 8 < 0$

16. approximately  $-0.66 \leq x \leq 2.45$

17. a)  $\{x \in \mathbb{R}, -1 \leq x \leq 0\}$ ,  $\{y \in \mathbb{R}, 0 \leq y \leq \frac{1}{2}\}$

b)  $\{x \in \mathbb{R}, x < -1, x > 1\}$ ,  $\{y \in \mathbb{R}, y > 0\}$

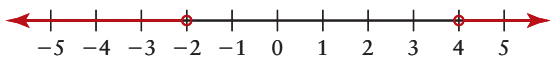
19. 15:34

20.  $2\sqrt{6}$

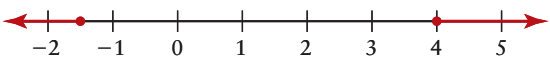
### 2.6 Solving Factorable Polynomial Inequalities Algebraically, pages 138–139

1. a)  $x \leq 2$  b)  $x > -\frac{5}{2}$  c)  $x \leq -\frac{1}{3}$  d)  $x < 1$  e)  $x < -2$  f)  $x \geq \frac{10}{3}$

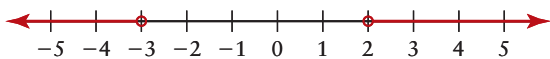
2. a)  $x < -2$  or  $x > 4$



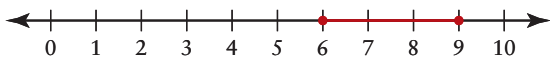
b)  $x \leq -\frac{3}{2}$  or  $x \geq 4$



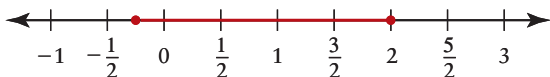
3. a)  $x < -3$  or  $x > 2$



b)  $6 \leq x \leq 9$



c)  $-\frac{1}{4} \leq x \leq 2$



4. a)  $-2 < x < -1$  or  $x > 3$  b)  $x \leq -7$  or  $\frac{1}{3} \leq x \leq 1$

c)  $x < -2.5$  or  $-\frac{2}{7} < x < 1$  d)  $-4 \leq x \leq -2$  or  $x \geq \frac{1}{3}$

5. a)  $x \leq 3$  or  $x \geq 5$  b)  $-3 < x < 5$  c)  $-\frac{2}{5} \leq x \leq \frac{4}{3}$

d)  $x < -2$  or  $1 < x < 3$  e)  $-\frac{3}{2} \leq x \leq -1$  or  $x \geq 1$

6. a) approximately  $x \geq -5.09$  b)  $x < -4$  or  $-3 \leq x \leq -2$

c)  $x \leq -1$  or  $\frac{2}{5} \leq x \leq 3$  d) true for all intervals

7. a)  $-5 \leq x \leq 1$  b)  $-2 < x < -\frac{1}{2}$  or  $x > 3$

c)  $-1 < x < -\frac{1}{2}$  or  $x > \frac{1}{2}$

d)  $\frac{-1 - \sqrt{17}}{2} \leq x \leq 1$  or  $x \geq \frac{-1 + \sqrt{17}}{2}$

8. 22 cm by 24 cm by 10 cm

9. after 10 years (in 2009)

11. 8

12. approximately  $x < 0.59$  or  $1 < x < 3.41$

13. a) approximately  $7 < n < 11$  or  $19 < n < 20$ , so between 7 and 11 years from today and between 19 and 20 years from today b) approximately  $12 < n < 18.6$ , so between 12 and 19 years from today c) Not valid beyond 20 years. 20 years from today the population will have fallen to 5560, and in the next year it would fall below 0, which is not possible.

14.  $x^4 - 76x^2 + 1156 \leq 0$ ,  $-x^4 + 76x^2 - 1156 \geq 0$

15.  $3\sqrt{13}$

16.  $y = \frac{4}{3}x - \frac{25}{3}$

### Chapter 2 Review, pages 140–141

1. a) i) 37 ii)  $\frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}$ ,  $x \neq 2$

b) i) -12 ii)  $\frac{12x^3 - 2x^2 + x - 11}{3x + 1} = 4x^2 - 2x + 1 - \frac{12}{3x + 1}$ ,  $x \neq -\frac{1}{3}$

c) i)  $\frac{27}{2}$  ii)  $\frac{-8x^4 - 4x + 10x^3 - x^2 + 15}{2x - 1}$   
 $= -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x - 1)}$ ,  $x \neq \frac{1}{2}$

2. a)  $k = -\frac{77}{27}$  b) 162

3.  $b = -34$

4. a)  $(x - 3)(x - 2)(x + 1)$  b)  $(x - 4)(x + 2)(3x + 1)$

c)  $(x - 3)(x - 1)(x + 6)(5x + 2)$

5. a)  $4(x - 2)(x + 1)(x + 2)$  b)  $(x - 2)(5x - 3)(5x + 3)$

c)  $x(x - 2)(x + 2)(2x + 5)$

6. a)  $(2x - 1)$  metres by  $(x + 3)$  metres by  $(x + 1)$  metres

b) 4 m by 2 m by 1 m

7.  $k = -2$

8.  $x = -4$  or  $x = -2$  or  $x = 3$

9. a)  $x = -4$  or  $x = 4$  b)  $x = \frac{1 - \sqrt{105}}{4}$  or  $x = \frac{1 + \sqrt{105}}{4}$

10. a)  $x = -1$  or  $x \neq -0.5$  or  $x \neq 0.8$

b)  $x \neq -0.7$  or  $x \neq 0.9$  or  $x \neq 8.8$

11.  $V = l(l - 5)(2l + 1)$ ; approximately 8.55 cm by 3.55 cm by 18.10 cm

12. B (different zeros)

13. a)  $y = k(x^3 - 4x^2 - x)$  b)  $y = -2(x^3 - 4x^2 - x)$

14.  $-3(x + 2)^2(x - 1)$

15. a)  $x \leq -4.2$  or  $x \geq 1.2$  b)  $-\frac{1}{2} < x < 3$  or  $x > 4$

c)  $x < -1.7$  or  $0.4 < x < 3.3$  d)  $x \leq -4$  or  $-\frac{1}{3} \leq x \leq 3$

e)  $x < -2.2$  or  $x > 2.2$

16. approximately between 0.8 s and 7.6 s and between 20 s and 23.6 s

17. a)  $-\frac{4}{5} < x < 4$  b)  $-\frac{3}{2} \leq x \leq \frac{2}{3}$  or  $x \geq 1$

c)  $x < -5$  or  $x > 5$

18. a)  $x \leq -\frac{7}{3}$  or  $x \geq \frac{1}{4}$  b)  $x \leq -4$  or  $\frac{1}{3} \leq x \leq \frac{3}{2}$

c) approximately  $x < -2.4$  or  $x > 4.3$

### Chapter 2 Practice Test, pages 142–143

1. C

2. C

3. D

4. a)  $\frac{x^3 - 4x^2 + 3x - 7}{x + 3} = x^2 - 7x + 24 - \frac{79}{x + 3}$

b)  $x \neq -3$  c)  $(x + 3)(x^2 - 7x + 24) - 79$

5. a)  $k = \frac{1}{2}$  b) 193

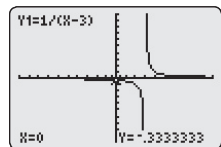
6. a)  $(x-4)(x-2)(x+1)$  b)  $(x-3)(x+2)(x+3)$   
 c)  $(x-2)(x+3)(x+4)$  d)  $(x-1)(x+2)(5x+2)$   
 e)  $(x+2)(x+3)(x+4)$  f)  $(x+1)(x+2)(x+3)(2x+1)$   
 7.  $x = -5$  or  $x = 3$  or  $x = -2$   
 8. a)  $x = 2$  b)  $x = -11$  or  $x = 11$  c)  $x = -5$  or  $x = 5$   
 d)  $x = -3$  or  $x = 3$  or  $x = -2$  or  $x = 5$   
 9. a)  $x = -2$  or  $x = -1$  b)  $x = -4$  or  $x = 1$  or  $x = 3$   
 c)  $x = -1.75$  or  $x = 1.5$  or  $x = 1.75$   
 d)  $x = -\frac{2}{3}$  or  $x = 0$  or  $x = \frac{3}{5}$  or  $x = \frac{2}{3}$   
 10. Answers may vary.  
 11. a)  $y = -\frac{1}{2}x(x+3)(2x+3)(x-2)$   
 b)  $x < -3, -\frac{3}{2} < x < 0, x > 2$   
 12. a)  $y = k(x^4 - 6x^3 - 17x^2 + 120x - 50)$   
 b)  $y = -\frac{2}{5}(x^4 - 6x^3 - 17x^2 + 120x - 50)$   
 13. a)  $V = x(20 - 2x)(18 - x)$  b) approximately 16.7 cm by 16.34 cm by 1.7 cm or 5.8 cm by 10.9 cm by 7.1 cm  
 c)  $V = k(20 - 2x)(18 - x)x$   
 14. a)  $x \leq -0.9$  or  $0.4 \leq x \leq 7.4$   
 b)  $-2.0 < x < -0.6$  or  $0.9 < x < 4.7$   
 15. a) approximately  $x < -3.6$  or  $-1.1 < x < 1.7$   
 b)  $-1.5 \leq x \leq -1$  or approximately  $x \leq -1.7$  or approximately  $x \geq 1.7$   
 16. a)  $-\frac{4}{3} < x < \frac{4}{3}$  b)  $x < 0$  c)  $x \leq -3$  or  $-\frac{5}{2} \leq x \leq 3$   
 d)  $x \leq -3$  or  $-1 \leq x \leq -\frac{1}{2}$  or  $x \geq 2$   
 17. a)  $V = x(32 - 2x)(40 - 2x)$   
 b) i)  $V = 2x(32 - 2x)(40 - 2x)$   
 ii)  $V = \frac{1}{2}x(32 - 2x)(40 - 2x)$  c) family of functions  
 d) approximately  $2 < x < 10.9$  or  $x > 23.1$

### CHAPTER 3

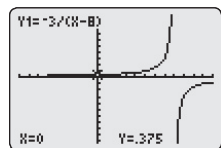
#### Prerequisite Skills, pages 146–147

1. Answers may vary. Sample answer: A line or curve that the graph approaches more and more closely. For  $f(x) = \frac{1}{x}$ , the vertical asymptote is  $x = 0$ .

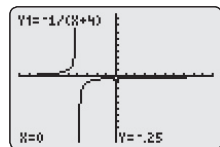
2. a)  $x = 3, y = 0$



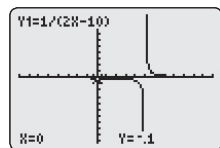
c)  $x = 8, y = 0$



b)  $x = -4, y = 0$



d)  $x = 5, y = 0$



3. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$  b)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 4\}$   
 c)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$  d)  $\{x \in \mathbb{R}, x \neq 0\}, \{y \in \mathbb{R}, y \neq 0\}$   
 e)  $\{x \in \mathbb{R}, x \neq 4\}, \{y \in \mathbb{R}, y \neq 0\}$  f)  $\{x \in \mathbb{R}, x \neq 0\}, \{y \in \mathbb{R}, y \neq 0\}$   
 4. a)  $-13$  b)  $\frac{1}{3}$  c)  $-\frac{2}{9}$  d)  $-1$  e)  $-13$  f)  $\frac{6}{5}$   
 5. a) 0.38 b) 0.71 c) 0.18 d) 0.38 e) 0.53 f) 0.35

6. a)  $(x+4)(x+3)$  b)  $(5x-2)(x-3)$  c)  $(3x+8)(2x-1)$   
 d)  $(x+1)(x+3)(x-2)$  e)  $(2x+1)^2(3x-2)$   
 f)  $(3x-4)(9x^2+12x+16)$   
 7. a) 8,  $-4$  b)  $-5, -1$  c)  $3, \frac{3}{2}$  d)  $-5, -\frac{1}{6}$  e)  $-7, \frac{1}{2}$  f) 6,  $-\frac{5}{3}$   
 8. a)  $-2 \pm \sqrt{2}$  b)  $\frac{-4 \pm \sqrt{14}}{2}$  c)  $\frac{5 \pm \sqrt{73}}{6}$  d) no  $x$ -intercepts  
 e)  $\frac{-4 \pm \sqrt{10}}{3}$  f)  $1 \pm 2\sqrt{2}$   
 9. a)  $x > 6$  b)  $x \leq \frac{11}{2}$  c)  $x > \frac{1}{2}$  d)  $x > -5$  e)  $x > -\frac{5}{2}$  f)  $x > -9$   
 10. a)  $-2 < x < 2$  b)  $x < -3$  or  $x > 6$  c)  $-\sqrt{13} < x < \sqrt{13}$   
 d)  $x < -5$  or  $x > 2$  e)  $-7 \leq x \leq -1$  f)  $x \leq -6$  or  $x \geq \frac{1}{2}$

#### 3.1 Reciprocal of a Linear Function, pages 153–155

1. a)

As $x \rightarrow$	$f(x) \rightarrow$
$2^+$	$+\infty$
$2^-$	$-\infty$
$+\infty$	0
$-\infty$	0

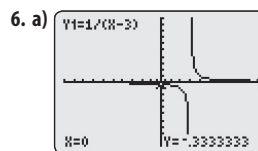
b)

As $x \rightarrow$	$f(x) \rightarrow$
$-5^+$	$+\infty$
$-5^-$	$-\infty$
$+\infty$	0
$-\infty$	0

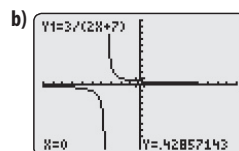
c)

As $x \rightarrow$	$f(x) \rightarrow$
$8^+$	$+\infty$
$8^-$	$-\infty$
$+\infty$	0
$-\infty$	0

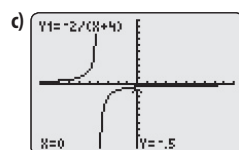
2. a) i)  $x = 2, y = 0$  ii)  $x = -3, y = 0$   
 b) i)  $y = \frac{1}{x-2}$  ii)  $y = \frac{1}{x+3}$   
 3. a) i)  $x = 5$  ii)  $y = 0$  iii)  $-\frac{1}{5}$  b) i)  $x = -6$  ii)  $y = 0$  iii)  $\frac{1}{3}$   
 c) i)  $x = 1$  ii)  $y = 0$  iii) 5 d) i)  $x = -7$  ii)  $y = 0$  iii)  $-\frac{1}{7}$   
 e) i)  $x = -3$  ii)  $y = 0$  iii)  $\frac{1}{12}$   
 5. a)  $y = \frac{1}{x-3}$  b)  $y = \frac{1}{x+3}$  c)  $y = \frac{1}{2x-1}$  d)  $y = -\frac{1}{x+4}$



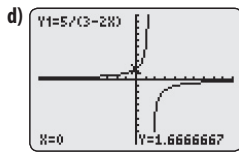
The slope is negative and decreasing for  $x < 3$ .  
 The slope is negative and increasing for  $x > 3$ .



The slope is negative and decreasing for  $x < -\frac{7}{2}$ .  
 The slope is negative and increasing for  $x < -\frac{7}{2}$ .

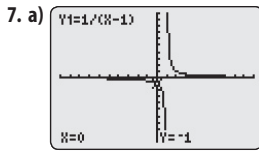


The slope is positive and increasing for  $x < -4$ .  
 The slope is positive and decreasing for  $x > -4$ .

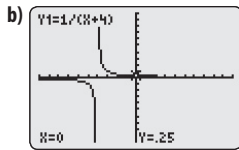


The slope is positive and increasing for  $x < \frac{3}{2}$ .

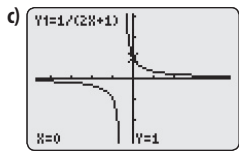
The slope is positive and decreasing for  $x > \frac{3}{2}$ .



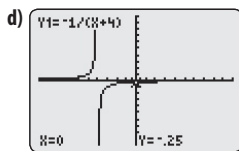
$\{x \in \mathbb{R}, x \neq 1\}, \{y \in \mathbb{R}, y \neq 0\}, x = 1, y = 0$



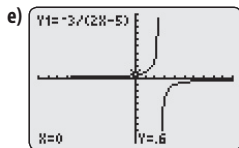
$\{x \in \mathbb{R}, x \neq -4\}, \{y \in \mathbb{R}, y \neq 0\}, x = 1, y = 0$



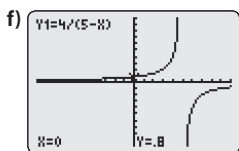
$\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}, \{y \in \mathbb{R}, y \neq 0\}, x = -\frac{1}{2}, y = 0$



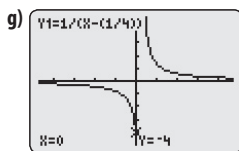
$\{x \in \mathbb{R}, x \neq -4\}, \{y \in \mathbb{R}, y \neq 0\}, x = -4, y = 0$



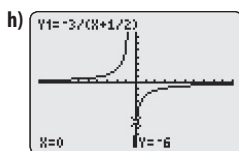
$\{x \in \mathbb{R}, x \neq \frac{5}{2}\}, \{y \in \mathbb{R}, y \neq 0\}, x = \frac{5}{2}, y = 0$



$\{x \in \mathbb{R}, x \neq 5\}, \{y \in \mathbb{R}, y \neq 0\}, x = 5, y = 0$



$\{x \in \mathbb{R}, x \neq \frac{1}{4}\}, \{y \in \mathbb{R}, y \neq 0\}, x = \frac{1}{4}, y = 0$

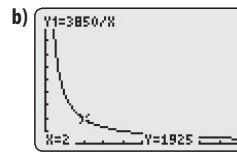


$\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}, \{y \in \mathbb{R}, y \neq 0\}, x = -\frac{1}{2}, y = 0$

8.  $y = \frac{1}{x-1}$

9.  $y = -\frac{1}{4x+4}$

10. a)  $t = \frac{3850}{v}$

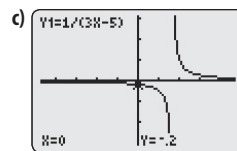
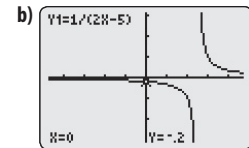
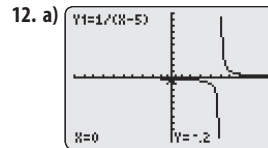


c) 7.7 h, or 7 h 42 min

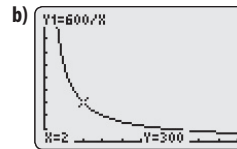
d) As the speed increases, the rate of change of the time decreases.

11. a) Answers may vary.

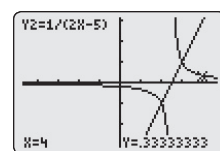
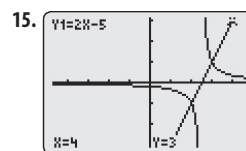
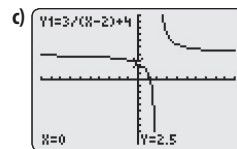
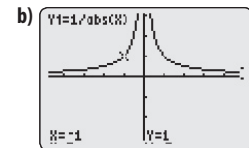
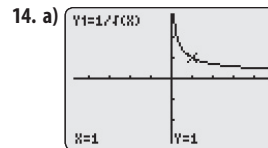
b) The equation of the asymptote is  $x = -\frac{2}{b}$ . When  $b = 1$ , the asymptote is  $x = -2$ . When  $b > 1$ ,  $-2 < -\frac{2}{b} < 0$ . When  $0 < b < 1$ ,  $-\frac{2}{b} < -2$ . When  $b < 0$ ,  $-\frac{2}{b} > 0$ .



13. a)  $F = \frac{600}{d}$



c) 300 N d) The force is halved.



Answers may vary. Sample answer: The reciprocal of the  $y$ -coordinates on either side of the  $x$ -intercept of  $y = 2x - 5$  are the  $y$ -coordinates of  $f(x) = \frac{1}{2x-5}$ .

16.  $x = \frac{yz}{y-z}, y \neq z, x \neq 0, y \neq 0, z \neq 0$

17. 14

18. E

### Extension, page 156

2. a) no; does not divide pixels evenly  
 b) yes; divides pixels evenly  
 3. Answers may vary. Sample answer:  $X_{\max} = 47$

### 3.2 Reciprocal of a Quadratic Function, pages 164–167

1. a)

As $x \rightarrow$	$f(x) \rightarrow$
$3^-$	$-\infty$
$3^+$	$+\infty$
$1^-$	$+\infty$
$1^+$	$-\infty$
$-\infty$	0
$+\infty$	0

b)

As $x \rightarrow$	$f(x) \rightarrow$
$-4^-$	$-\infty$
$-4^+$	$-\infty$
$5^-$	$-\infty$
$5^+$	$+\infty$
$-\infty$	0
$+\infty$	0

c)

As $x \rightarrow$	$f(x) \rightarrow$
$-6^-$	$-\infty$
$-6^+$	$-\infty$
$-\infty$	0
$+\infty$	0

2. a)  $x = 4; \{x \in \mathbb{R}, x \neq 4\}$   
 b)  $x = 2, x = -7; \{x \in \mathbb{R}, x \neq 2, x \neq -7\}$  c)  $\{x \in \mathbb{R}\}$   
 d)  $x = -5, x = 5; \{x \in \mathbb{R}, x \neq 5, x \neq -5\}$   
 e)  $x = 3, x = 1; \{x \in \mathbb{R}, x \neq 3, x \neq 1\}$   
 f)  $x = -4, x = -3; \{x \in \mathbb{R}, x \neq -4, x \neq -3\}$   
 g)  $x = -2, x = \frac{4}{3}; \{x \in \mathbb{R}, x \neq -2, x \neq \frac{4}{3}\}$  h)  $\{x \in \mathbb{R}\}$

3. a)

Interval	$x < 1$	$x > 1$
Sign of $f(x)$	+	+
Sign of Slope	+	-
Change in Slope	+	-

b)

Interval	$x < -2$	$-2 < x < 1$	$x = 1$	$1 < x < 4$	$x > 4$
Sign of $f(x)$	+	-	-	-	+
Sign of Slope	+	+	0	-	-
Change in Slope	+	-	-	+	-

c)

Interval	$x < -3$	$-3 < x < 0$	$x = 0$	$0 < x < 3$	$x > 3$
Sign of $f(x)$	-	+	+	+	-
Sign of Slope	-	-	0	+	+
Change in Slope	-	+	+	+	-

d)

Interval	$x < -4$	$x > -4$
Sign of $f(x)$	-	-
Sign of Slope	-	+
Change in Slope	+	-

4. a)  $y = \frac{1}{(x-1)^2}$  b)  $y = \frac{1}{(x+2)(x-4)}$  c)  $y = -\frac{1}{x^2-9}$

d)  $y = -\frac{1}{(x+4)^2}$

5. a) i)  $\{x \in \mathbb{R}, x \neq -3, x \neq 3\}$  ii)  $x = 3, x = -3, y = 0$

iii) y-intercept  $-\frac{1}{9}$



v)

Interval	$x < -3$	$-3 < x < 0$	$x = 0$	$0 < x < 3$	$x > 3$
Sign of $f(x)$	+	-	-	-	+
Sign of Slope	+	+	0	-	-
Change in Slope	+	-	-	-	+

vi)  $\{y \in \mathbb{R}, y \neq 0\}$  b) i)  $\{x \in \mathbb{R}, x \neq -3, x \neq 5\}$

ii)  $x = -3, x = 5, y = 0$  iii) y-intercept  $-\frac{1}{15}$



v)

Interval	$x < -3$	$-3 < x < 1$	$x = 1$	$1 < x < 5$	$x > 5$
Sign of $f(x)$	+	-	-	-	+
Sign of Slope	+	+	0	-	-
Change in Slope	+	-	-	-	+

vi)  $\{y \in \mathbb{R}, y \neq 0\}$  c) i)  $\left\{x \in \mathbb{R}, x \neq \frac{-5 \pm \sqrt{109}}{2}\right\}$

ii)  $x = \frac{-5 \pm \sqrt{109}}{2}, y = 0$  iii) y-intercept  $\frac{1}{21}$



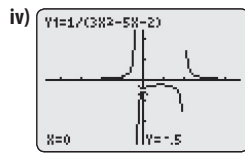
v)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < \frac{-5 - \sqrt{109}}{2}$	-	-	-
$\frac{-5 - \sqrt{109}}{2} < x < -2.5$	+	-	+
$x = -2.5$	+	0	+
$-2.5 < x < \frac{-5 + \sqrt{109}}{2}$	+	+	+
$x > \frac{-5 + \sqrt{109}}{2}$	-	+	-

vi)  $\{y \in \mathbb{R}, y \neq 0\}$

d) i)  $\{x \in \mathbb{R}, x \neq 2, x \neq -\frac{1}{3}\}$  ii)  $x = 2, x = -\frac{1}{3}, y = 0$

iii) y-intercept  $-\frac{1}{2}$

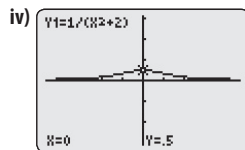


v)

Interval	$x < -\frac{1}{3}$	$-\frac{1}{3} < x < \frac{5}{6}$	$x = \frac{5}{6}$	$\frac{5}{6} < x < 2$	$x > 2$
Sign of $f(x)$	+	-	-	-	+
Sign of Slope	+	+	0	-	-
Change in Slope	+	-	-	-	+

vii)  $\{x \in \mathbb{R}, y \neq 0\}$

e) i)  $\{x \in \mathbb{R}\}$  ii)  $y = 0$  iii) y-intercept  $\frac{1}{2}$



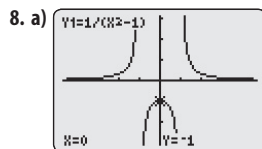
v)

Interval	$x < 0$	$x = 0$	$x > 0$
Sign of $f(x)$	+	+	+
Sign of Slope	+	0	-
Change in Slope	+	-	+

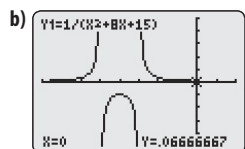
vi)  $\{y \in \mathbb{R}, 0 < y < \frac{1}{2}\}$

6. Answers may vary. a) 0 b) 0.009 c) 0.011 d) 1.250 e) 0

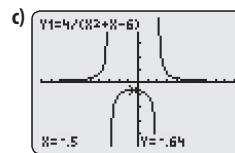
7. a)  $\{x \in \mathbb{R}, x \neq 0\}$ ;  $\{y \in \mathbb{R}, y > 0\}$ ; asymptotes  $x = 0, y = 0$ ; no x- or y-intercepts; for  $x < 0$ , the function is positive and increasing (positive slope); for  $x > 0$ , the function is positive and decreasing (negative slope) b)  $\{x \in \mathbb{R}, x \neq 1\}$ ;  $\{y \in \mathbb{R}, y > 0\}$ ; asymptotes  $x = 1, y = 0$ ; y-intercept 1; for  $x < 1$ , the function is positive and increasing (positive slope); for  $x > 1$ , the function is positive and decreasing (negative slope) c)  $\{x \in \mathbb{R}, x \neq -2\}$ ;  $\{y \in \mathbb{R}, y > 0\}$ ; asymptotes  $x = -2, y = 0$ ; y-intercept  $\frac{1}{4}$ ; for  $x < -2$ , the function is positive and increasing (positive slope); for  $x > -2$ , the function is positive and decreasing (negative slope)



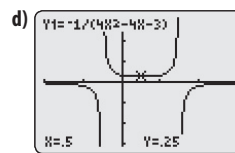
8. a) increasing for  $x < -1$  and  $-1 < x < 0$ , decreasing for  $0 < x < 1$  and  $x > 1$



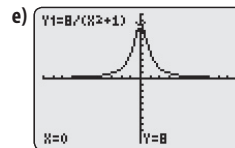
b) increasing for  $x < -5$  and  $-5 < x < -4$ , decreasing for  $-4 < x < -3$  and  $x > -3$



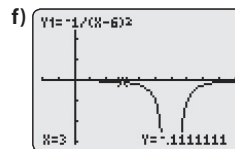
increasing for  $x < -3$  and  $-3 < x < -\frac{1}{2}$ , decreasing for  $-\frac{1}{2} < x < 2$  and  $x > 2$



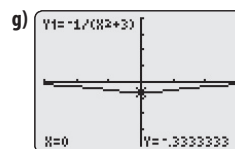
increasing for  $\frac{1}{2} < x < \frac{3}{2}$  and  $x > \frac{3}{2}$ , decreasing for  $x < -\frac{1}{2}$  and  $-\frac{1}{2} < x < \frac{1}{2}$



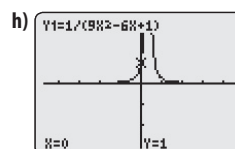
increasing for  $x < 0$ , decreasing for  $x > 0$



increasing for  $x > 6$ , decreasing for  $x < 6$

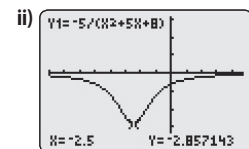
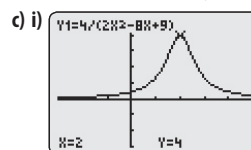


increasing for  $x > 0$ , decreasing for  $x < 0$



increasing for  $x < \frac{1}{3}$ , decreasing for  $x > \frac{1}{3}$

9. a), b) Answers may vary.



10. Answers may vary. Sample answers: a)  $f(x)$  and  $g(x)$  will have the same shape reflected in the x-axis.

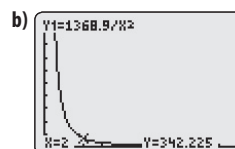
b)  $h(x) = \frac{2}{x^2 - 9}$  is a vertical stretch of  $k(x) = \frac{1}{x^2 - 9}$  by a factor of 2.

c)  $m(x)$  and  $n(x)$  will have the same shape but different asymptotes.

11. Answers may vary. Sample answers: a)  $y = \frac{1}{x^2 + x - 6}$

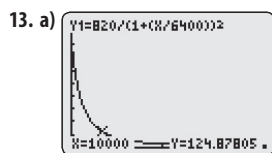
b)  $y = \frac{1}{x^2 + 2}$  c)  $y = -\frac{1}{(x + 3)^2}$

12. a)  $I \doteq \frac{1368.9}{d^2}$

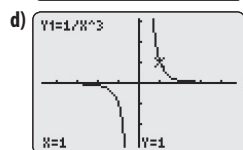
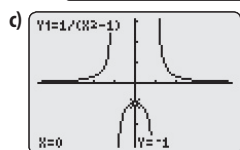
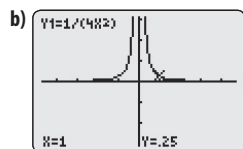
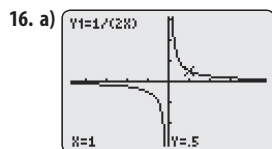
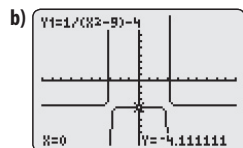
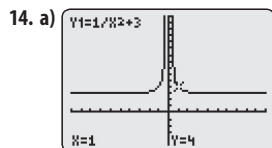


c)  $I \doteq 1368.9$ ; rate of change is approximately  $-2737.8$ .

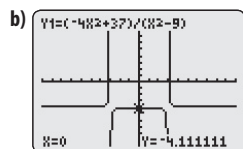
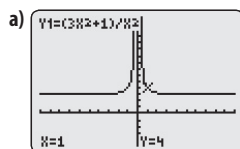




- b) i) approximately 817.4 N ii) approximately 310.5 N  
 c)  $b \geq 51\,554.5$  km



17. a) symmetric about the origin  
 b) symmetric about the y-axis  
 18. Explanations may vary.



19. Answers may vary. Sample answer:  $y = \frac{1}{(x-a)(x-b)}$

20.  $a < -27$  or  $a > \frac{1}{2}$

21. 7

22. C

### 3.3 Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$ , pages 174–176

1. a)  $x = 7, \{x \in \mathbb{R}, x \neq 7\}$  b)  $x = -5, \{x \in \mathbb{R}, x \neq -5\}$

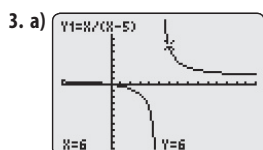
c)  $x = -8, \{x \in \mathbb{R}, x \neq -8\}$  d)  $x = \frac{1}{3}, \{x \in \mathbb{R}, x \neq \frac{1}{3}\}$

e)  $x = -\frac{9}{4}, \{x \in \mathbb{R}, x \neq -\frac{9}{4}\}$  f)  $x = 5, \{x \in \mathbb{R}, x \neq 5\}$

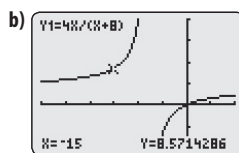
2. a)  $y = 1, \{y \in \mathbb{R}, y \neq 1\}$  b)  $y = 3, \{y \in \mathbb{R}, y \neq 3\}$

c)  $y = 1, \{y \in \mathbb{R}, y \neq 1\}$  d)  $y = \frac{5}{2}, \{y \in \mathbb{R}, y \neq \frac{5}{2}\}$

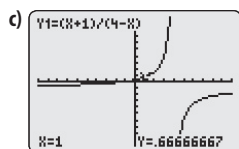
e)  $y = -1, \{y \in \mathbb{R}, y \neq -1\}$  f)  $y = 2, \{y \in \mathbb{R}, y \neq 2\}$



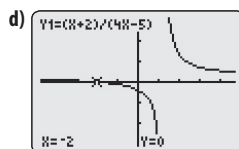
Interval	$x < 0$	$0 < x < 5$	$x > 5$
Sign of $f(x)$	+	-	+
Sign of Slope	-	-	-
Change in Slope	-	-	+



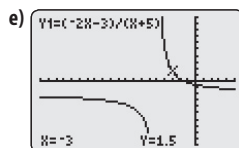
Interval	$x < -8$	$-8 < x < 0$	$x > 0$
Sign of $f(x)$	+	-	+
Sign of Slope	+	+	+
Change in Slope	+	-	-



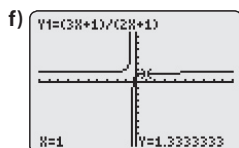
Interval	$x < -1$	$-1 < x < 4$	$x > 4$
Sign of $f(x)$	-	+	-
Sign of Slope	+	+	+
Change in Slope	+	+	-



Interval	$x < -2$	$-2 < x < \frac{5}{4}$	$x > \frac{5}{4}$
Sign of $f(x)$	+	-	+
Sign of Slope	-	-	-
Change in Slope	+	-	-



Interval	$x < -5$	$-5 < x < -1.5$	$x > -1.5$
Sign of $f(x)$	+	+	+
Sign of Slope	+	-	-
Change in Slope	+	-	+



Interval	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < -\frac{1}{3}$	$x > -\frac{1}{3}$
Sign of $f(x)$	+	-	+
Sign of Slope	+	+	+
Change in Slope	+	+	-

4. a) i)  $m_{3.5} = -24, m_{20} = -0.02$  ii)  $m_{2.5} = -24, m_{-20} = -0.02$

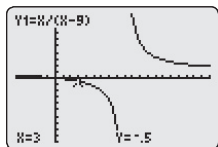
- b) The function is decreasing for  $x < 3$  and increasing for  $x > 3$ .

5. a)  $y = \frac{1}{2}$  ii)  $y = -\frac{5}{2}$  iii)  $y = 2$

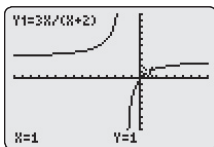
b) Answers may vary. Sample answer: The horizontal asymptote is equal to the coefficient of  $x$  in the numerator divided by the coefficient of  $x$  in the denominator.

c)  $y = \frac{a}{c}$

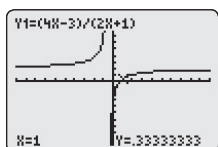
6. a)  $y = 1, x = 9,$   
 $\{x \in \mathbb{R}, x \neq 9\},$   
 $\{y \in \mathbb{R}, y \neq 1\}$



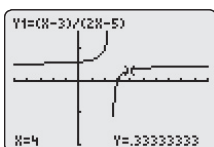
b)  $y = 3, x = -2,$   
 $\{x \in \mathbb{R}, x \neq -2\},$   
 $\{y \in \mathbb{R}, y \neq 3\}$



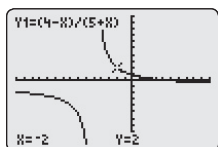
c)  $y = 2, x = -\frac{1}{2},$   
 $\{x \in \mathbb{R}, x \neq -\frac{1}{2}\},$   
 $\{y \in \mathbb{R}, y \neq 2\}$



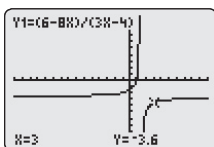
d)  $y = \frac{1}{2}, x = \frac{5}{2},$   
 $\{x \in \mathbb{R}, x \neq \frac{5}{2}\},$   
 $\{y \in \mathbb{R}, y \neq \frac{1}{2}\}$



e)  $y = -1, x = -5,$   
 $\{x \in \mathbb{R}, x \neq -5\},$   
 $\{y \in \mathbb{R}, y \neq -1\}$



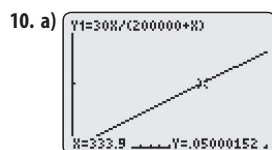
f)  $y = -\frac{8}{3}, x = \frac{4}{3},$   
 $\{x \in \mathbb{R}, x \neq \frac{4}{3}\},$   
 $\{y \in \mathbb{R}, y \neq -\frac{8}{3}\}$



7. a)  $y = \frac{2x-3}{x-3}$  b)  $y = \frac{x-4}{x+1}$

8.  $y = \frac{x+4}{x-2}$

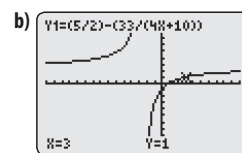
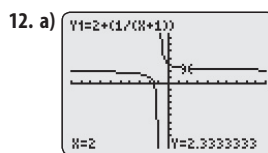
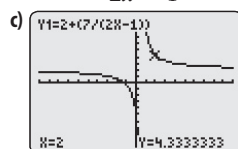
9.  $y = \frac{5x-3}{2x+1}$



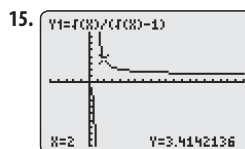
b) The amount of pollutant levels off at 30 g/L.

c) after approximately 333.9 min

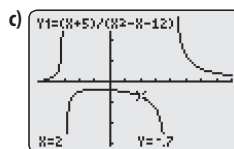
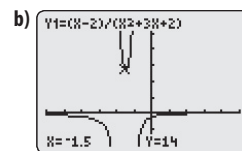
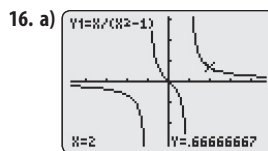
11. a)  $2 + \frac{7}{2x-1}$  b) Answers may vary.



14. Answers may vary. Sample answer: As the mass of the club increases, the rate of change of the initial velocity decreases.



asymptotes  $y = 1, x = 1; \{x \in \mathbb{R}, x > 0, x \neq 1\}, \{y \in \mathbb{R}, y \leq 0, y > 1\};$  y-intercept 0; for  $0 < x < 1, f(x)$  is negative and decreasing and the slope is negative and decreasing; for  $x > 1, f(x)$  is positive and decreasing and the slope is negative and increasing. Comparison: Answers may vary.



Common features:  
Answers may vary.

17. Answers may vary. Sample answer: When the degree of the polynomial in the numerator is greater than the degree of the polynomial in the denominator, you can expect to get an oblique asymptote.

18. A

19. a) quotient  $x$ , remainder  $-2$  b)  $x - \frac{2}{x}$

c) i)  $y = x - 1, x = 2$  ii)  $y = \frac{x+3}{2}, x = 2$

iii) discontinuous at  $x = -3$  (linear)

### 3.4 Solve Rational Equations and Inequalities, pages 183–185

1. a)  $-1$  b)  $-4, 3$  c)  $\frac{3}{2}$  d)  $0$

2. a)  $x = \frac{10}{3}$  b)  $x = 4$  or  $x = -2$  c)  $x = \frac{11}{3}$  d)  $x = 5$  or  $x = -1$

e)  $x = -34$  f)  $x = 2$

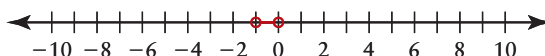
3. a)  $x = 0$  or  $x \doteq 6.71$  b)  $x \doteq -0.27$  or  $x \doteq -18.73$

c)  $x \doteq 4.34$  or  $x \doteq 2.47$  or  $x \doteq 0.19$  d)  $x \doteq 1.28$  or  $x \doteq -1.28$

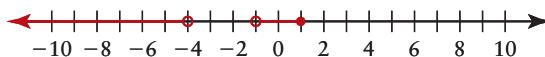
4. a)  $x < 3$  or  $x > 7$



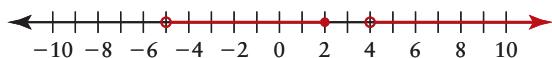
b)  $-1 < x < 0$



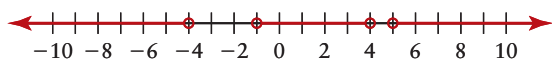
c)  $x < -4$  or  $-1 < x \leq 1$



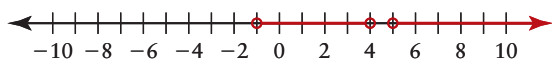
d)  $-5 < x \leq 2$  or  $x > 4$



e)  $x < -4$  or  $-1 < x < 4$  or  $x > 5$



f)  $0 < x < 3$  or  $x > 6$



5. a)  $x < -7$  or  $-2 < x < 1$  or  $x > 5$  b)  $-3 < x < \frac{1}{2}$

c)  $-6 < x < -5$  or  $-1 \leq x \leq 4$  d)  $x < \frac{1}{2}$  or  $\frac{2}{3} \leq x \leq 2$  or  $x > 5$

6. Answers may vary. Sample answer:  $\frac{2x-3}{(x-3)(x+5)} = 0$

7.  $x < -4$  or  $-1 < x < 0$  or  $x > 2$ ; points of intersection  $(-4, 0)$  and  $(0, 0)$

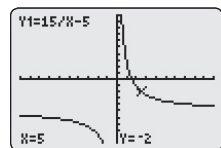
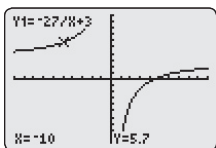
8.  $-5 < x < 0$  or  $3 < x < 7$

9. a)  $x = \frac{1}{3}$  b)  $x = \frac{-3 \pm \sqrt{14}}{5}$  c)  $x = 2$  or  $x = 6$

d)  $\{x \in \mathbb{R}, x \neq 1\}$  e)  $x = \frac{-3 \pm 3\sqrt{2}}{2}$  f) no solution

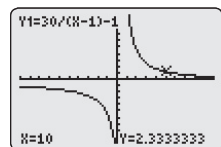
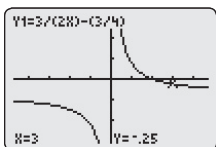
10. a)  $x < 0$  or  $x > 9$

b)  $x < 0$  or  $x > 3$



c)  $0 < x < 2$

d)  $x < 1$  or  $x > 31$



11.  $x < -\frac{25}{2}$  or  $x > 5$

12.  $x < -7$  or  $-3 < x < -2$  or  $x > 5$  versus  $-7 < x < -3$  or  $-2 < x < 5$

13.  $x < -5$  or  $\frac{7}{13} \leq x < 4$  versus  $-1 < x \leq \frac{7}{13}$  or  $x > 3$

14. a)  $\frac{1}{2a} + \frac{1}{2b} = \frac{1}{x}$  b)  $\frac{40}{3}$  c)  $b = \frac{2}{3}$

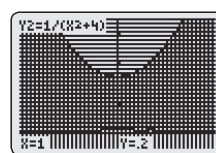
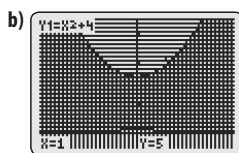
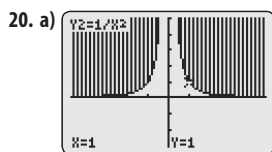
15. a) i) 3600 lux ii) 2.25 lux b) i) 141.4 m ii)  $0 < d \leq 2\sqrt{5}$

16.  $2 < I < \frac{5}{2}$

18. a)  $l \doteq 31.26$  cm,  $w \doteq 0.74$  cm

b)  $x = y = \frac{\sqrt{2}}{2}$  or  $x = y = \frac{-\sqrt{2}}{2}$

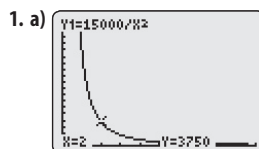
19. a)  $x = 2$  b)  $x > 2$



21. a)  $\frac{3}{x-1} + \frac{2}{x+3}$  b)  $\frac{27}{5(x-3)} + \frac{8}{5(x+2)}$

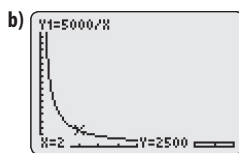
c)  $\frac{1}{x+2} + \frac{5}{x-3} - \frac{3}{(x-3)^2}$

### 3.5 Making Connections With Rational Functions and Equations, pages 189–191



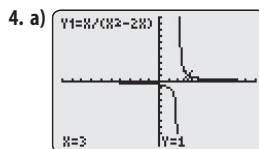
b) The light intensity is less.  
c) When  $d$  is close to 0, the light intensity is very great.

2. a)  $V = \frac{5000}{p}$

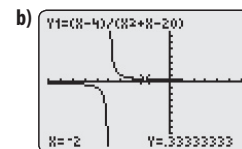


c) The volume is halved.

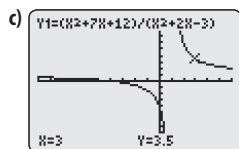
3. a)  $x = 1$  b)  $-1 < x < \frac{1}{3}$



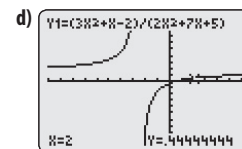
discontinuous at  $(0, -\frac{1}{2})$



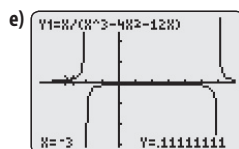
discontinuous at  $(4, \frac{1}{9})$



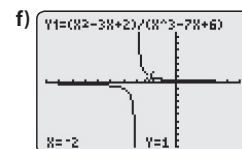
discontinuous at  $(-3, -\frac{1}{4})$



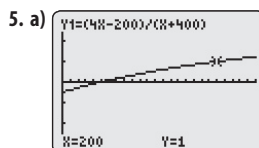
discontinuous at  $(-1, -\frac{5}{3})$



discontinuous at  $(0, -\frac{1}{12})$



discontinuous at  $(1, \frac{1}{4})$  and  $(2, \frac{1}{5})$

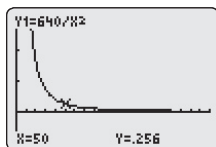


b) Answers may vary.  
Sample answer: Average profit is modelled by  $\frac{P(x)}{x} =$  slope of secant.

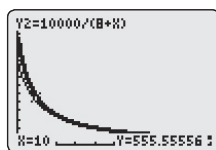
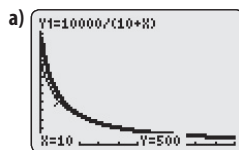
c) The average profit is the greatest when  $x = 200$ .

d)  $9.18 \times 10^{-4}$

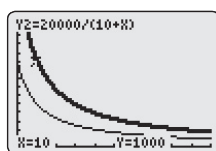
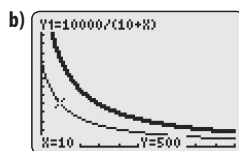
6. a)  $R = \frac{0.64l}{d^2}$  b)  $R = \frac{640}{d^2}$



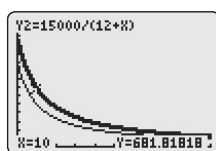
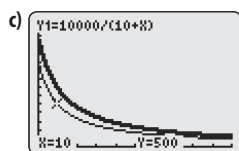
7. Answers may vary. Sample answers:



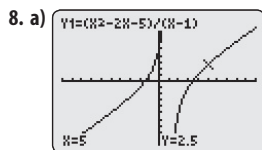
The cost is just slightly greater per person than the original model. The cost decreases at a greater rate at first.



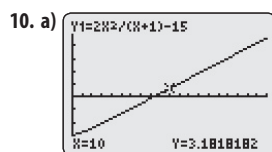
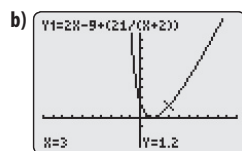
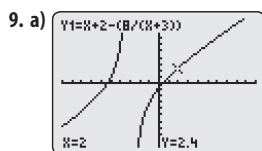
The cost is much greater per person. The gap between the graphs decreases as the number of passengers increases. The cost decreases at a slower rate.



The cost per person is greater. As the number of passengers increases, the cost per person decreases and the graphs get closer. The cost decreases at a slightly slower rate.

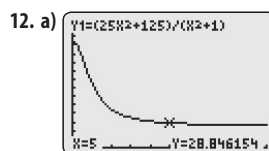
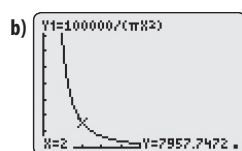


b) a slanting asymptote c)  $x - 1 - \frac{6}{x - 1}$  d)  $y = x - 1$



b) approximately 8.39 h c) approximately 5.85 h

11. a)  $b = \frac{100\,000}{\pi r^2}$

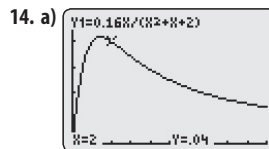


b) The systolic pressure decreases and gets closer to 25.

c) The rate of change decreases until  $t \approx 0.58$  s and then increases gradually, getting closer to 0.

d) rate of change of  $P(t)$  at  $t = 5$  is  $-1.48$ ; for  $R(t)$  it is  $0.84$

13. 1



b) The curve increases to reach a maximum concentration of  $C = 0.0418$  mg/cm<sup>3</sup> when  $t \approx 1.414$  min and then gradually decreases to  $C$  as time increases close to 0.

15. increasing for  $0 < R < 0.40$

16. False because the function is discontinuous at point (2, 4).

17. A

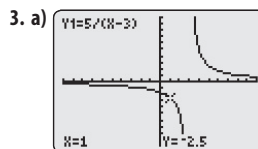
18.  $x = \frac{\pi}{3} + 2k\pi, k = 0, \pm 1, \pm 2, \pm 3, \dots$

(i.e.,  $\dots, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, \dots$ )

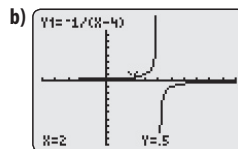
### Chapter 3 Review, pages 192–193

1. a)  $x = 2, y = 0$  b)  $x = -7, y = 0$  c)  $x = 5, y = 0$

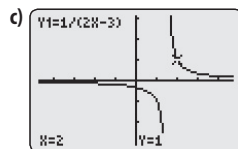
2. a)  $y = \frac{2}{x - 1}$  b)  $y = \frac{1}{x + 4}$



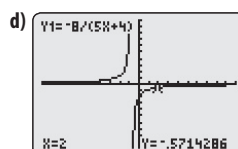
$\{x \in \mathbb{R}, x \neq 3\}, \{y \in \mathbb{R}, y \neq 0\}, -\frac{5}{3}, x = 3, y = 0$



$\{x \in \mathbb{R}, x \neq 4\}, \{y \in \mathbb{R}, y \neq 0\}, \frac{1}{4}, x = 4, y = 0$

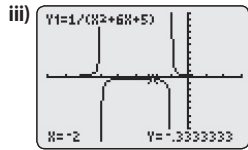


$\{x \in \mathbb{R}, x \neq \frac{3}{2}\}, \{y \in \mathbb{R}, y \neq 0\}, -\frac{1}{3}, x = \frac{3}{2}, y = 0$

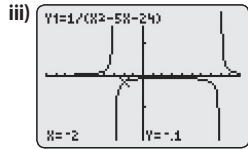


$\{x \in \mathbb{R}, x \neq -\frac{4}{5}\}, \{y \in \mathbb{R}, y \neq 0\}, -2, x = -\frac{4}{5}, y = 0$

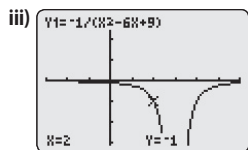
4. a)  $x = 3, x = -4, \{x \in \mathbb{R}, x \neq -4, x \neq 3\}$   
 b)  $x = -3, \{x \in \mathbb{R}, x \neq -3\}$   
 c)  $x = -6, x = -2, \{x \in \mathbb{R}, x \neq -6, x \neq -2\}$   
 5. a) i)  $x = -5, x = -1, y = 0$  ii)  $\frac{1}{5}$



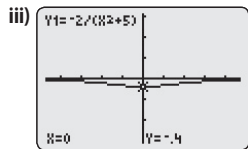
- iv) increasing for  $x < -5$  and  $-5 < x < -3$ , decreasing for  $-3 < x < -1$  and  $x > -1$   
 v)  $\{x \in \mathbb{R}, x \neq -5, x \neq -1\}, \{y \in \mathbb{R}, y > 0, y \leq -\frac{1}{4}\}$   
 b) i)  $x = 8, x = -3, y = 0$  ii)  $-\frac{1}{24}$



- iv) increasing for  $x < -3$  and  $-3 < x < 2.5$ , decreasing for  $2.5 < x < 8$  and  $x > 8$   
 v)  $\{x \in \mathbb{R}, x \neq 8, x \neq -3\}, \{y \in \mathbb{R}, y > 0, y \leq -\frac{4}{121}\}$   
 c) i)  $x = 3, y = 0$  ii)  $-\frac{1}{9}$



- iv) increasing for  $x > 3$ , decreasing for  $x < 3$   
 v)  $\{x \in \mathbb{R}, x \neq 3\}, \{y \in \mathbb{R}, y < 0\}$   
 d) i)  $y = 0$  ii)  $-\frac{2}{5}$



- iv) increasing for  $x > 0$ , decreasing for  $x < 0$   
 v)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, -\frac{2}{5} \leq y < 0\}$

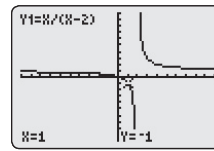
6.

Interval	Sign of Slope	Change in Slope
$x < -\frac{5}{2}$	+	+
$-\frac{5}{2} < x < -\frac{3}{4}$	+	-
$x = -\frac{3}{4}$	0	-
$-\frac{3}{4} < x < 1$	-	-
$x > 1$	-	+

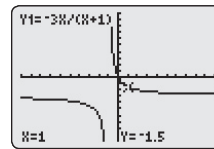
7.  $y = \frac{1}{(x+4)(x-5)}$

8. a)  $y = 1$  b)  $y = -2$  c)  $y = 1$   
 9. a)  $x = 2, y = 1, \{x \in \mathbb{R}, x \neq 2\}, \{y \in \mathbb{R}, y \neq 1\}$ , y-intercept 0; for  $x < 0$ ,  $f(x)$  is positive and decreasing and the slope is negative and decreasing; for  $0 < x < 2$ ,  $f(x)$  is negative and

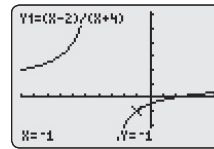
decreasing and the slope is negative and decreasing; for  $x > 2$ ,  $f(x)$  is positive and decreasing and the slope is negative and increasing



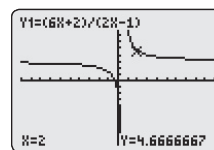
- b)  $x = -3, y = -1, \{x \in \mathbb{R}, x \neq -1\}, \{y \in \mathbb{R}, y \neq -3\}$   
 y-intercept 0; for  $x < -1$ ,  $f(x)$  is negative and decreasing and the slope is negative and decreasing; for  $-1 < x < 0$ ,  $f(x)$  is positive and decreasing and the slope is negative and increasing; for  $x > 0$ ,  $f(x)$  is negative and decreasing and the slope is negative and increasing



- c)  $x = -4, y = 1, \{x \in \mathbb{R}, x \neq -4\}, \{y \in \mathbb{R}, y \neq 1\}$ , y-intercept  $-\frac{1}{2}$ , x-intercept 2; for  $x < -4$ ,  $f(x)$  is positive and increasing and the slope is positive and increasing; for  $-4 < x < 2$ ,  $f(x)$  is negative and increasing and the slope is positive and decreasing; for  $x > 2$ ,  $f(x)$  is positive and increasing and the slope is positive and decreasing

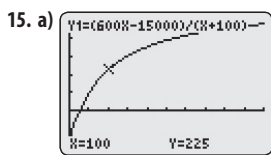


- d)  $x = \frac{1}{2}, y = 3, \{x \in \mathbb{R}, x \neq \frac{1}{2}\}, \{y \in \mathbb{R}, y \neq 3\}$ ;  
 y-intercept  $-2$ , x-intercept  $-\frac{1}{3}$ ; for  $x < -\frac{1}{3}$ ,  $f(x)$  is positive and decreasing and the slope is negative and decreasing; for  $-\frac{1}{3} < x < \frac{1}{2}$ ,  $f(x)$  is negative and decreasing and the slope is negative and decreasing; for  $x > \frac{1}{2}$ ,  $f(x)$  is positive and decreasing and the slope is negative and increasing



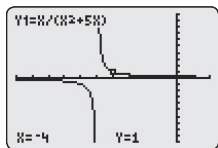
10.  $f(x) = \frac{4x-1}{3x+2}$

11. a)  $x = \frac{15}{2}$  b)  $x = -9$  or  $x = 3$   
 12. a)  $x = 0$  or  $x \approx 0.86$  b)  $x \approx 40.88$  or  $x \approx 0.12$  c)  $x \approx 1.64$   
 13. a)  $x < -5$  or  $x > -\frac{7}{2}$   
 b)  $-3 < x < -2$  or  $x \geq 1$   
 c)  $x < -4$  or  $-2 < x < 5$  or  $x > 6$   
 d)  $-7 < x < -5$  or  $x > -\frac{5}{3}$   
 14. a)  $-4 < x < -1$  or  $2 < x < 3$   
 b)  $x < -8$  or  $x > -\frac{1}{2}$  and  $x \neq 3$

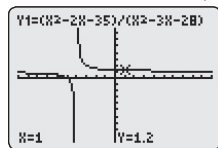


- b) Profit increases as sales increase.  
 c) The rate of change of the profit at 100 t is 1.875 and approximately 0.208 at 500 t, so the rate of change is decreasing.

16. a) discontinuous at  $(0, \frac{1}{5})$



b) discontinuous at  $(7, \frac{12}{11})$



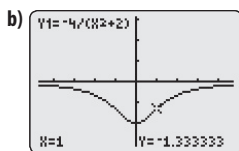
### Chapter 3 Practice Test, pages 194–195

1. C  
 2. B  
 3. A

4. Answers may vary. Sample answers:

a)  $y = \frac{2}{x+2}$  b)  $y = \frac{6}{(x+4)(x-3)}$

5. a) i)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, -2 \leq y < 0\}$  ii) y-intercept  $-2$   
 iii)  $y = 0$  iv) decreasing for  $x < 0$ , increasing for  $x > 0$



6. Yes;  $\frac{1}{f(x)}$  will always have an asymptote at  $y = 0$ .

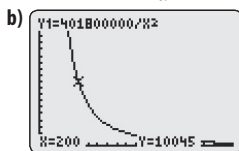
7. a)  $x = -\frac{14}{5}$  b)  $x = 3$

8. a)  $x < -\frac{3}{2}$ ,  $x > -\frac{7}{8}$  b)  $x < -1$  or  $2 < x < 5$

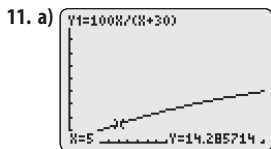
9. a) Answers may vary. Sample answer:  $y = \frac{-x+2}{2(x+1)}$

b) Yes. Sample answer:  $y = \frac{-2x+4}{4(x+1)}$

10. a)  $g = \frac{40180000}{d^2}$

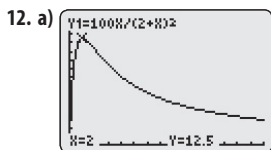


c)  $d \doteq 8183.3$  km



b)  $\{t \in \mathbb{R}, t \geq 0\}$ ,  $\{P \in \mathbb{R}, 0 \leq P < 100\}$

c) The percent lost can get close to 100%, but not equal to 100%.



- b) The power output increases from  $0 \Omega$  to  $2 \Omega$ . The power decreases from  $2 \Omega$  to  $20 \Omega$ .  
 c) The power is constant at  $R = 2$  (not changing).

13. Answers may vary. Sample answer:  $x = 0$ ,  $y = 0$ , slopes increasing and decreasing faster as  $n$  increases.

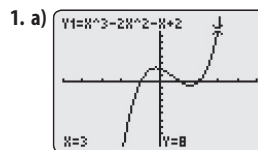
$n$  even:

- For  $x < 0$ ,  $f(x)$  is positive and the slope is positive and increasing.
- For  $x > 0$ ,  $f(x)$  is positive and the slope is negative and increasing.

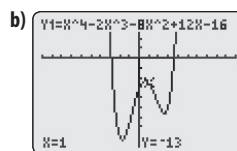
$n$  odd:

- For  $x < 0$ ,  $f(x)$  is negative and the slope is negative and decreasing.
- For  $x > 0$ ,  $f(x)$  is positive and the slope is negative and increasing.

### Chapter 1 to 3 Review, pages 196–197



x-intercepts  $-1$ ,  $1$ , and  $2$ ; y-intercept  $2$



x-intercepts approximately  $-2.88$  and  $3.63$ ; y-intercept  $-16$

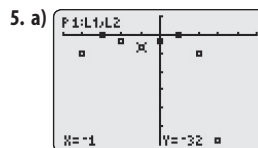
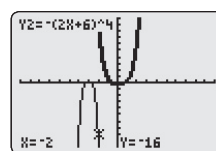
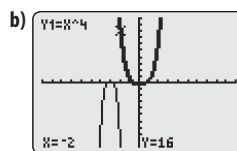
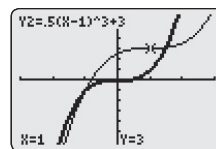
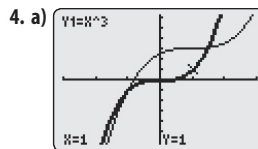
2. a) The graph extends from quadrant 2 to quadrant 1, thus, as  $x \rightarrow -\infty$ ,  $y \rightarrow \infty$ , and as  $x \rightarrow \infty$ ,  $y \rightarrow \infty$ .

The graph is not symmetric.

b) The graph extends from quadrant 3 to quadrant 1, thus, as  $x \rightarrow -\infty$ ,  $y \rightarrow -\infty$ , and as  $x \rightarrow \infty$ ,  $y \rightarrow \infty$ .

The graph has point symmetry about  $(0.55, 0)$ .

3. a) i)  $-61$  ii)  $-37$  b)  $-49$  approximates the instantaneous rate of change at  $x = 2$ .



b) 4 c)  $y = -2(x-1)^2(x+3)^2$

d) Answers may vary. Sample answer: Reflects and stretches the graph. Also, since the function has even degree, a negative leading coefficient means the graph extends from quadrant 3 to quadrant 4 and has at least one maximum point.

6. a)  $-4$  b)  $12$  c) local minimum

7. a) maximum approximately (12.25, 9.64), minima approximately (3.14, -14.16), (26.61, -70.80)  
 b) between (3.14, -14.16) and (12.25, 9.64) approximately 2.61; between (12.25, 9.64), and (26.61, -70.80), approximately -5.60 c)  $x = 32$

8. Answers may vary. Sample answers:

$$y = 2x(x+7)(x-3)^2; y = -\frac{1}{3}x(x+7)(x-3)^2$$

9.  $y = k(x-2)^2(x+5)$ . Answers may vary. Sample answers:  $y = 2(x-2)^2(x+5)$ , y-intercept 10;  $y = -3(x-2)^2(x+5)$ , y-intercept -60

10. a)  $4x^3 - 5x^2 + 6x + 2 = (2x+1)\left(2x^2 - \frac{7}{2}x + \frac{19}{4}\right) - \frac{11}{4}$ ,  $x \neq -\frac{1}{2}$

b)  $3x^4 - 5x^2 - 28 = (x-2)(3x^3 + 6x^2 + 7x + 14)$ ,  $x \neq 2$

11. a) 38 b)  $\frac{97}{9}$

12. a) No. b) Yes.

13.  $k = -\frac{13}{2}$

14. a)  $(x-3)(x^2+3x+9)$  b)  $(x-2)(2x^2+8x+3)$

15. a)  $x = -4$  or  $x = 1$  or  $x = 5$

b)  $x = -3$  or  $x = \frac{-4 - \sqrt{31}}{5}$  or  $x = \frac{-4 + \sqrt{31}}{5}$

16. a)  $x \leq 1$  or  $x \geq 6$  b)  $x < -3$  or  $-2 < x < 2$

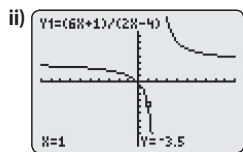
17. from 0 min to 10 min

18. A

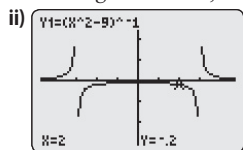
19. a)  $f(x) \rightarrow 0$  b)  $f(x) \rightarrow 0$  c)  $f(x) \rightarrow \infty$  d)  $f(x) \rightarrow -\infty$

20.  $\{x \in \mathbb{R}, x \neq -1\}$ ,  $\{y \in \mathbb{R}, y \neq 1\}$

21. a) i)  $\{x \in \mathbb{R}, x \neq 2\}$ ,  $\{y \in \mathbb{R}, y \neq 3\}$ ; x-intercept  $-\frac{1}{6}$ , y-intercept  $-\frac{1}{4}$ ; asymptotes  $x = 2$ ,  $y = 3$ ; negative slope  $x < 2$ ,  $x > 2$ ; decreasing  $x < 2$ ; increasing  $x > 2$



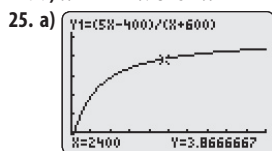
b) i)  $\{x \in \mathbb{R}, x \neq -3, x \neq 3\}$ ,  $\{y \in \mathbb{R}, y \neq 0\}$ ; no x-intercept, y-intercept  $-\frac{1}{9}$ ; asymptotes  $x = -3$ ,  $x = 3$ ,  $y = 0$ ; positive slope  $x < -3$ ,  $-3 < x < 0$ ; negative slope  $0 < x < 3$ ,  $x > 3$ ; decreasing  $0 < x < 3$ ,  $-3 < x < 0$ ; increasing  $x < -3$ ,  $x > 3$



22.  $f(x) = \frac{3x+6}{x-1}$

23. a)  $x = -2.2$  b)  $x \approx 2.15$

24. a)  $x < -2.75$  or  $x > -2$  b)  $-1 \leq x < 1$  or  $x > 2$



b)  $\{x \in \mathbb{R}, x \geq 0\}$ ,  $\{P(x) \in \mathbb{R}, -\frac{2}{3} \leq P(x) < 5\}$

c) The profit is always less than \$5000.

26. Since  $t$  represents time,  $t \geq 0$ ;  $t \neq 10$  because the denominator cannot be zero.

## CHAPTER 4

### Prerequisite Skills, pages 200–201

1. a)  $\cos \theta = \frac{4}{5}$ ,  $\tan \theta = \frac{3}{4}$  b)  $\sin \theta = -\frac{12}{13}$ ,  $\tan \theta = -\frac{12}{5}$

c)  $\sin x = \frac{7}{25}$ ,  $\cos x = -\frac{24}{25}$  d)  $\cos x = -\frac{15}{17}$ ,  $\tan x = \frac{8}{15}$

2. a) 0.2588 b) 0.5592 c) 3.7321 d) 0.9848 e) -0.9205

f) 2.7475 g) -0.8480 h) 0.9781

3. a)  $41^\circ$  b)  $65^\circ$  c)  $83^\circ$  d)  $117^\circ$

4. a)  $\frac{1}{2}$  b) 3 c)  $\frac{5}{3}$  d)  $\frac{2}{\sqrt{3}}$

5. a)  $\sec x = \frac{5}{3}$ ,  $\cot x = \frac{3}{4}$  b)  $\csc \theta = -\frac{13}{5}$ ,  $\cot \theta = -\frac{12}{5}$

c)  $\csc x = \frac{25}{7}$ ,  $\sec x = -\frac{25}{24}$  d)  $\sec \theta = -\frac{17}{8}$ ,  $\cot \theta = \frac{8}{15}$

6. a) 1.7434 b) -1.2361 c) 2.1445 d) 1.1792 e) -1.2690

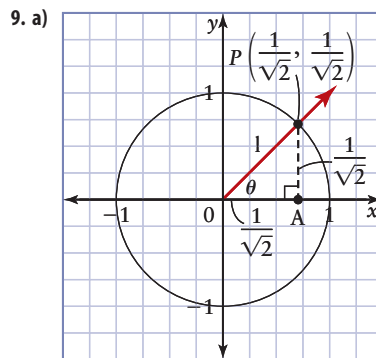
f) 1.0724 g) -1.5890 h) 1.0038

7. a)  $53^\circ$  b)  $54^\circ$  c)  $18^\circ$  d)  $139^\circ$

8. a)  $\sin \theta = \frac{1}{2}$ ,  $\cos \theta = \frac{\sqrt{3}}{2}$ ,  $\tan \theta = \frac{1}{\sqrt{3}}$

b)  $\sin \theta = \frac{1}{\sqrt{2}}$ ,  $\cos \theta = \frac{1}{\sqrt{2}}$ ,  $\tan \theta = 1$

c)  $\sin \theta = \frac{\sqrt{3}}{2}$ ,  $\cos \theta = \frac{1}{2}$ ,  $\tan \theta = \sqrt{3}$



b)  $x = \frac{1}{\sqrt{2}}$ ,  $y = \frac{1}{\sqrt{2}}$  c)  $\csc 45^\circ = \sqrt{2}$ ,  $\sec 45^\circ = \sqrt{2}$ ,  $\cot 45^\circ = 1$

10.  $\sin 135^\circ = \frac{1}{\sqrt{2}}$ ,  $\cos 135^\circ = -\frac{1}{\sqrt{2}}$ ,  $\tan 135^\circ = -1$ ,

$\csc 135^\circ = \sqrt{2}$ ,  $\sec 135^\circ = -\sqrt{2}$ ,  $\cot 135^\circ = -1$ ,

$\sin 225^\circ = -\frac{1}{\sqrt{2}}$ ,  $\cos 225^\circ = -\frac{1}{\sqrt{2}}$ ,  $\tan 225^\circ = 1$ ,

$\csc 225^\circ = -\sqrt{2}$ ,  $\sec 225^\circ = -\sqrt{2}$ ,  $\cot 225^\circ = 1$ ,

$\sin 315^\circ = -\frac{1}{\sqrt{2}}$ ,  $\cos 315^\circ = \frac{1}{\sqrt{2}}$ ,  $\tan 315^\circ = -1$ ,

$\csc 315^\circ = -\sqrt{2}$ ,  $\sec 315^\circ = \sqrt{2}$ ,  $\cot 315^\circ = -1$

11. a) 5 b) 13 c) 17 d) 10

12. a)  $a^2 + 2ab + b^2$  b)  $c^2 - d^2$  c)  $6x^2 - xy - 2y^2$

d)  $\sin^2 x + 2 \sin x \cos y + \cos^2 y$

### 4.1 Radian Measure, pages 202–210

1. a)  $\frac{\pi}{3}$  b)  $\frac{\pi}{2}$  c)  $\frac{2\pi}{3}$  d)  $\frac{5\pi}{6}$

2. a)  $\frac{\pi}{12}$  b)  $\frac{\pi}{18}$  c)  $\frac{\pi}{24}$  d)  $\frac{\pi}{36}$

3. a)  $\frac{\pi}{2}$  b)  $\frac{3\pi}{4}$  c)  $\pi$  d)  $\frac{5\pi}{4}$

4. a)  $\frac{\pi}{8}$  b)  $\frac{\pi}{12}$  c)  $\frac{\pi}{20}$  d)  $\frac{\pi}{60}$

5. a)  $\frac{2\pi}{9}$  b)  $\frac{\pi}{18}$  c)  $\frac{7\pi}{4}$  d)  $\frac{7\pi}{6}$  e)  $\frac{5\pi}{3}$  f)  $\frac{5\pi}{12}$   
 6. a) 0.40 b) 0.89 c) 1.43 d) 2.23 e) 4.19 f) 5.76  
 7. a)  $36^\circ$  b)  $20^\circ$  c)  $75^\circ$  d)  $50^\circ$  e)  $135^\circ$  f)  $270^\circ$   
 8. a)  $134.1^\circ$  b)  $179.9^\circ$  c)  $301.9^\circ$  d)  $431.4^\circ$  e)  $39.0^\circ$  f)  $98.5^\circ$   
 9. 118.75 cm  
 10. a)  $720^\circ/\text{s}$  b)  $4\pi$  rad/s  
 11.  $\frac{\pi}{5}$ ,  $\frac{2\pi}{5}$ ,  $\frac{2\pi}{5}$   
 12. Answers may vary depending on speed.  
 13. a) 0.000 291 b) 1862 m c) Answers may vary.  
 14. b) 8 km  
 15. 0.009 053 rad,  $0.5^\circ$   
 16. 0.5 rad,  $28.6^\circ$   
 17.  $400\pi$  rad/s, approximately 1256.6 rad/s  
 18. a)  $\frac{80\pi}{3}$  m b) 83.8 m  
 19. a) It must follow the rotation of Earth. b) 24 h  
 c) 0.000 023 $\pi$  rad/s d) It is the same.  
 20. a) 100 grad b)  $\frac{3\pi}{4}$

22. Using  $v = \sqrt{\frac{G \times M_{\text{Earth}}}{R}}$ , where  $G = 6.673 \times 10^{-11}$

$\text{N}\cdot\text{m}^2/\text{kg}^2$ ,  $M_{\text{Earth}} = 5.98 \times 10^{24}$  kg, and  $R$  is the radius of orbit for the satellite, the orbital speed of a geostationary satellite is approximately 3073 m/s.

23. Using the modern definition of a nautical mile, 1852 m, the knots are approximately 15.4 m apart.

24. the same

25. a)  $A = \frac{1}{2}r^2\theta$  b) 45.24  $\text{cm}^2$

26. a)  $A\left(1, \frac{\pi}{3}\right)$ ,  $B\left(2, \frac{\pi}{4}\right)$ ,  $C\left(2, \frac{7\pi}{4}\right)$ ,  $D\left(2, \frac{3\pi}{2}\right)$

b) i)  $\left(\sqrt{2}, \frac{\pi}{4}\right)$  ii) (5, 2.21) iii)  $\left(5, \frac{3\pi}{2}\right)$

#### 4.2 Trigonometric Ratios and Special Angles, pages 211–219

1. a) i) 0.4226 ii) 0.3090 iii)  $-2.1445$  iv) 0.2588 b) i) 0.4223  
 ii) 0.3087 iii)  $-2.1452$  iv) 0.2586 c) The degree measures are approximately the same as the radian measures.  
 2. a) i) 0.9356 ii)  $-0.8187$  iii)  $-0.0918$  iv) 0.0076  
 b) i) 0.9336 ii)  $-0.8192$  iii)  $-0.0875$  iv) 0.0000  
 c) The degree measures are approximately the same as the radian measures.  
 3. a) 0.7071 b) 0.9010 c)  $-0.5774$  d) 0.4142  
 4. a) 3.8637 b) 1.6243 c)  $-0.6745$  d)  $-2.6695$   
 5. a) 1.2123 b)  $-3.7599$  c) 14.5955 d) 1.0582  
 6. a)  $-2.0000$  b) 1.5270 c)  $-0.3249$  d)  $-2.7475$

7. a)  $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ ,  $\cos \frac{2\pi}{3} = -\frac{1}{2}$ ,  $\tan \frac{2\pi}{3} = -\sqrt{3}$

b)  $\sin \frac{5\pi}{6} = \frac{1}{2}$ ,  $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ ,  $\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

c)  $\sin \frac{3\pi}{2} = -1$ ,  $\cos \frac{3\pi}{2} = 0$ ,  $\tan \frac{3\pi}{2} = \text{undefined}$

d)  $\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$ ,  $\cos \frac{7\pi}{4} = \frac{1}{\sqrt{2}}$ ,  $\tan \frac{7\pi}{4} = -1$

8. a)  $\sin \frac{7\pi}{6} = -\frac{1}{2}$ ,  $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ ,  $\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$ ,

$\csc \frac{7\pi}{6} = -2$ ,  $\sec \frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$ ,  $\cot \frac{7\pi}{6} = \sqrt{3}$

b)  $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ ,  $\cos \frac{4\pi}{3} = -\frac{1}{2}$ ,  $\tan \frac{4\pi}{3} = \sqrt{3}$ ,

$\csc \frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$ ,  $\sec \frac{4\pi}{3} = -2$ ,  $\cot \frac{4\pi}{3} = \frac{1}{\sqrt{3}}$

c)  $\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ ,  $\cos \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ ,  $\tan \frac{5\pi}{4} = 1$ ,

$\csc \frac{5\pi}{4} = -\sqrt{2}$ ,  $\sec \frac{5\pi}{4} = -\sqrt{2}$ ,  $\cot \frac{5\pi}{4} = 1$

d)  $\sin \pi = 0$ ,  $\cos \pi = -1$ ,  $\tan \pi = 0$ ,  $\csc \pi = \text{undefined}$ ,  
 $\sec \pi = -1$ ,  $\cot \pi = \text{undefined}$

9. a)  $20(\sqrt{2} - 1)$  m b)  $20(\sqrt{3} - \sqrt{2})$  m c) 8.3 m horizontally,  
 6.4 m vertically

10. a)  $(20\sqrt{2} - 30)$  m

b) The kite moves farther from Sarah, since the horizontal distance at  $\frac{\pi}{3}$  is now greater than at  $\frac{\pi}{4}$ .

c)  $(30\sqrt{3} - 20\sqrt{2})$  m; the altitude increases since the vertical distance of the kite at  $\frac{\pi}{3}$  has increased.

d) 1.7 m horizontally, 23.7 m vertically

11. a)  $\frac{\sqrt{2}}{2}$  b) 2

12. a)  $\frac{\sqrt{6}}{6}$  b) 2

13. a)  $30\sqrt{2}$  m b)  $15\sqrt{6}$  m

14. a)  $\frac{\pi}{2}$  b)  $\frac{2\pi}{3}$  c) 9:00 d) 11:00 e)  $\frac{5\pi}{4}$

15. b) 0.500 $\pi$  radians d) The values are approximately the same.

16. a) 0 b) 1

17. a) 1 b) 0

18. a) 0 b) undefined

21. a)  $\sin(150 \text{ grads}) = \frac{1}{\sqrt{2}}$ ,  $\cos(150 \text{ grads}) = -\frac{1}{\sqrt{2}}$ ,

$\tan(150 \text{ grads}) = -1$ ,  $\csc(150 \text{ grads}) = \sqrt{2}$ ,

$\sec(150 \text{ grads}) = -\sqrt{2}$ ,  $\cot(150 \text{ grads}) = -1$

b) Answers may vary.

22. d)  $0.00 \leq x \leq 0.30$  e)  $0.000 \leq x \leq 0.100$

23. Answers may vary.

24. A

25. B

#### 4.3 Equivalent Trigonometric Expressions, pages 220–227

1.  $\sin \frac{\pi}{6} = \cos \left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

2.  $\cos \frac{\pi}{4} = \sin \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

3.  $-\sin \frac{\pi}{6} = \cos \left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

4.  $-\csc \frac{\pi}{4} = \sec \left(\frac{\pi}{2} + \frac{\pi}{4}\right)$

5.  $\frac{5\pi}{14}$

6.  $\frac{\pi}{18}$

7.  $\frac{2\pi}{9}$

8.  $\frac{3\pi}{7}$

9. a) 0.6549 b) 0.6549

10. a) 0.8391 b)  $-0.8391$

11. 0.12

12. 0.93

13. 2.32

14. 2.91

15.  $\sin(\pi - x) = \sin x$ ,  $\cos(\pi - x) = -\cos x$ ,

$\tan(\pi - x) = -\tan x$ ,  $\csc(\pi - x) = \csc x$ ,

$\sec(\pi - x) = -\sec x$ ,  $\cot(\pi - x) = -\cot x$



16.  $\sin(\pi + x) = -\sin x$ ,  $\cos(\pi + x) = -\cos x$ ,  
 $\tan(\pi + x) = \tan x$ ,  $\csc(\pi + x) = -\csc x$ ,  
 $\sec(\pi + x) = -\sec x$ ,  $\cot(\pi + x) = \cot x$

17.  $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$ ,  $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$ ,

$\tan\left(\frac{3\pi}{2} - x\right) = \cot x$ ,  $\csc\left(\frac{3\pi}{2} - x\right) = -\sec x$ ,

$\sec\left(\frac{3\pi}{2} - x\right) = -\csc x$ ,  $\cot\left(\frac{3\pi}{2} - x\right) = \tan x$

18.  $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$ ,  $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$ ,

$\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$ ,  $\csc\left(\frac{3\pi}{2} + x\right) = -\sec x$ ,

$\sec\left(\frac{3\pi}{2} + x\right) = \csc x$ ,  $\cot\left(\frac{3\pi}{2} + x\right) = -\tan x$

19.  $\sin(2\pi - x) = -\sin x$ ,  $\cos(2\pi - x) = \cos x$ ,

$\tan(2\pi - x) = -\tan x$ ,  $\csc(2\pi - x) = -\csc x$ ,

$\sec(2\pi - x) = \sec x$ ,  $\cot(2\pi - x) = -\cot x$

20. Answers may vary.  
 21. Answers may vary. Sample answer:  $-\cos\frac{21\pi}{26}$

22. a)  $r = \frac{v^2}{g} \tan\left(\frac{\pi}{2} - \theta\right) = \frac{v^2}{g} \cot \theta = \frac{v^2}{g \tan \theta}$  b) 255 m

24. Answers may vary. Sample answer:  $\frac{\pi}{16}$

25. Answers may vary. Sample answer:  $\frac{\pi}{12}$

26. Answers may vary.

28. a) i)  $\left(1, \frac{\pi}{3}\right)$ ,  $\left(-1, -\frac{2\pi}{3}\right)$  ii)  $\left(5, -\frac{\pi}{6}\right)$ ,  $\left(-5, \frac{5\pi}{6}\right)$

b) i)  $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$  ii)  $(-4, 0)$

c) i)  $\frac{\pi}{6} + 2\pi k$ ,  $k \in \mathbb{Z}$  ii)  $-\frac{\pi}{3} + 2\pi k$ ,  $k \in \mathbb{Z}$

4.4 Compound Angle Formulas, pages 228–235

1. a)  $\sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$ ;  $\frac{\sqrt{3}}{2}$  b)  $\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$ ;  $\frac{1}{2}$

c)  $\cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$ ;  $\frac{1}{2}$  d)  $\cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$ ;  $\frac{\sqrt{3}}{2}$

2. a)  $\sin\left(\frac{3\pi}{5} + \frac{\pi}{15}\right)$ ;  $\frac{\sqrt{3}}{2}$  b)  $\sin\left(\frac{7\pi}{5} - \frac{\pi}{15}\right)$ ;  $-\frac{\sqrt{3}}{2}$

c)  $\cos\left(\frac{2\pi}{9} + \frac{5\pi}{18}\right)$ ; 0 d)  $\cos\left(\frac{10\pi}{9} - \frac{5\pi}{18}\right)$ ;  $-\frac{\sqrt{3}}{2}$

3. a)  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$  b)  $\frac{1 - \sqrt{3}}{2\sqrt{2}}$  c)  $\frac{-1 + \sqrt{3}}{2\sqrt{2}}$  d)  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

4. a)  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$  b)  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

5. a)  $\frac{-1 - \sqrt{3}}{2\sqrt{2}}$  b)  $\frac{-1 + \sqrt{3}}{2\sqrt{2}}$

6. a)  $\frac{1 - \sqrt{3}}{2\sqrt{2}}$  b)  $\frac{1 - \sqrt{3}}{2\sqrt{2}}$

7. a)  $\frac{-\sqrt{3} - 1}{2\sqrt{2}}$  b)  $\frac{1 + \sqrt{3}}{2\sqrt{2}}$

8. a)  $\cos x = \frac{4}{5}$  b)  $\sin y = \frac{12}{13}$

9. a)  $\frac{63}{65}$  b)  $-\frac{33}{65}$  c)  $-\frac{16}{65}$  d)  $\frac{56}{65}$

10. a)  $\cos x = -\frac{12}{13}$  b)  $\sin y = \frac{4}{5}$

11. a)  $-\frac{33}{65}$  b)  $\frac{63}{65}$  c)  $-\frac{56}{65}$  d)  $-\frac{16}{65}$

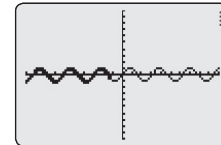
12.  $\sin 2\theta = \sin(\theta + \theta)$   
 $= \sin \theta \cos \theta + \cos \theta \sin \theta$   
 $= 2 \sin \theta \cos \theta$

13.  $\cos 2x = \cos(x + x)$   
 $= \cos x \cos x - \sin x \sin x$   
 $= \cos^2 x - \sin^2 x$

15. a)  $\frac{527}{625}$  b)  $-\frac{336}{625}$  c) 2.86

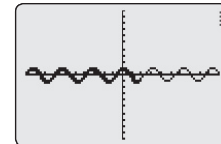
16. For question 12:

```
Plot1 Plot2 Plot3
V1 sin(2X)
V2 sin(X)cos(X)
V3 =
V4 =
V5 =
V6 =
```



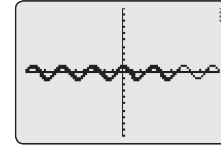
For question 13:

```
Plot1 Plot2 Plot3
V1 cos(2X)
V2 cos(X)^2 - sin(X)^2
V3 =
V4 =
V5 =
V6 =
```



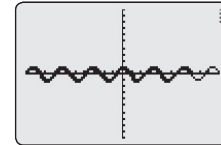
For question 14a):

```
Plot1 Plot2 Plot3
V1 cos(2X)
V2 1 - 2sin(X)^2
V3 =
V4 =
V5 =
V6 =
V7 =
```



For question 14b):

```
Plot1 Plot2 Plot3
V1 cos(2X)
V2 2cos(X)^2 - 1
V3 =
V4 =
V5 =
V6 =
V7 =
```



17. a)  $b_1 = 12 \sin x$

18. a)  $66.5^\circ$ ; Answers may vary. The Sun is not seen at all at this latitude.

b)  $-23.5^\circ$ ; Answers may vary. The negative sign represents a latitude in the southern hemisphere. The Sun appears directly overhead at noon.

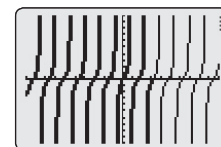
20. a)  $\tan(x + y) = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

c) Both sides of the equation equal  $-\frac{\sqrt{3}}{3}$ .

21. b) Both sides of the equation equal  $\frac{\sqrt{3}}{3}$ .

22. a)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

```
Plot1 Plot2 Plot3
V1 tan(2X)
V2 2tan(X)/(1-tan(X)^2)
V3 =
V4 =
V5 =
V6 =
```



c) Both sides of the formula equal approximately 1.7036.

23. a) Both sides of the formula equal  $\sqrt{3}$ .

b)  $\sin x - \sin y = 2 \sin\left(\frac{x - y}{2}\right) \cos\left(\frac{x + y}{2}\right)$

25. 0.71 rad

27. a)

$\theta$	0.01	0.05	0.10	0.15	0.25	0.35
$\theta - \frac{\theta^2}{6}$	0.01000	0.04998	0.09983	0.14944	0.24740	0.34285
$\sin \theta$	0.01000	0.04998	0.09983	0.14944	0.24740	0.34290

b)

$\theta$	0.01	0.05	0.10	0.15	0.25	0.35
$1 - \frac{\theta^2}{2}$	0.99995	0.99875	0.99500	0.98875	0.96875	0.93875
$\cos \theta$	0.99995	0.99875	0.99500	0.98877	0.96891	0.93937

28. a)  $\sin \theta = \theta$  b)  $\cos \theta = 1$  c)  $\tan \theta = \theta$

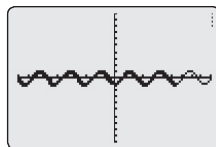
#### 4.5 Prove Trigonometric Identities, pages 236–241

9. b)

```

Plot1 Plot2 Plot3
V1=1-2cos(X)^2
V2=sin(X)cos(X)
V3=tan(X)-1/tan(X)
V4=
V5=

```



14. Answers may vary.

17. a) Yes, the graphs appear to be the same. b) identity

18. a) Answers may vary. Graphs are different.

b) While the left side results in both positive and negative values, the right side is restricted to positive values only.

20. a) Yes, the graphs appear to be the same. b) identity

$$22. \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

#### Chapter 4 Review, pages 244–245

1. a) 0.58 b) 2.41 c) 4.40 d) 6.06

2. a)  $71.0^\circ$  b)  $161.6^\circ$  c)  $273.9^\circ$  d)  $395.9^\circ$

3. a)  $\frac{5\pi}{12}$  b)  $\frac{\pi}{9}$  c)  $\frac{\pi}{15}$  d)  $\frac{\pi}{20}$

4. a)  $72^\circ$  b)  $80^\circ$  c)  $105^\circ$  d)  $110^\circ$

5. a)  $72^\circ/\text{s}$  b)  $\frac{2\pi}{5}$  rad/s

6. a), b)

Revolutions per Minute	16 rpm	$33\frac{1}{3}$ rpm	45 rpm	78 rpm
Degrees per Second	$96^\circ/\text{s}$	$200^\circ/\text{s}$	$270^\circ/\text{s}$	$468^\circ/\text{s}$
Radians per Second	$\frac{8\pi}{15}$ rad/s	$\frac{10\pi}{9}$ rad/s	$\frac{3\pi}{2}$ rad/s	$\frac{39\pi}{15}$ rad/s

$$7. \sin \frac{4\pi}{11} \approx 0.9096, \cos \frac{4\pi}{11} \approx 0.4154, \tan \frac{4\pi}{11} \approx 2.1897,$$

$$\csc \frac{4\pi}{11} \approx 1.0993, \sec \frac{4\pi}{11} \approx 2.4072, \cot \frac{4\pi}{11} \approx 0.4567$$

$$8. \text{ a) } 2 \text{ b) } \frac{\sqrt{2} + 1}{\sqrt{2}}$$

$$9. \frac{15\sqrt{3}}{4} \text{ m}$$

$$10. \frac{3\pi}{14}$$

$$11. \frac{\pi}{18}$$

$$12. \text{ a) } 5.6713; \cot \frac{\pi}{18} = \cot \left( \frac{\pi}{2} - \frac{4\pi}{9} \right) = \tan \frac{4\pi}{9}$$

$$\text{ b) } 5.6713; \tan \frac{13\pi}{9} = \tan \left( \frac{3\pi}{2} - \frac{\pi}{18} \right) = \cot \frac{\pi}{18}$$

$$13. \frac{17\pi}{22}$$

$$14. \text{ a) } \sin \left( \frac{5\pi}{12} + \frac{\pi}{4} \right); \frac{\sqrt{3}}{2} \text{ b) } \sin \left( \frac{5\pi}{12} - \frac{\pi}{4} \right); \frac{1}{2}$$

$$\text{ c) } \cos \left( \frac{5\pi}{12} + \frac{\pi}{4} \right); -\frac{1}{2} \text{ d) } \cos \left( \frac{5\pi}{12} - \frac{\pi}{4} \right); \frac{\sqrt{3}}{2}$$

$$15. \text{ a) } \cos x = \frac{3}{5} \text{ b) } \sin y = \frac{24}{25} \text{ c) } \frac{4}{5}$$

$$16. \text{ a) } \frac{527}{625} \text{ b) } \frac{336}{625}$$

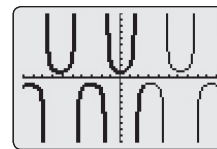
$$17. \frac{-1 - \sqrt{3}}{2\sqrt{2}}$$

19. b)

```

Plot1 Plot2 Plot3
V1=1/cos(X)
V2=(2*cos(X)*sin(2X)-sin(X)*cos(2X))/sin(2X)
V3=
V4=
V5=

```



21. a) No; the graphs are not the same for all values.

b) Rewrite  $3x$  as  $2x + x$ . Then, use the addition formula for cosine to expand  $\cos(2x + x)$ . Next, apply the appropriate double angle formulas and simplify.

22. Not an identity. Let  $x = 0$ ; L.S.  $\neq$  R.S.

#### Chapter 4 Practice Test, pages 246–247

1. B

2. C

3. C

4. D

5. B

6. C

7. A

8. a)  $13^\circ/\text{day}$ ,  $0.23$  rad/day b) 88 412 km/day

$$9. \frac{\sqrt{3}}{2}$$

$$10. \text{ a) } \frac{80}{\sqrt{6}} \text{ m b) } 32.7 \text{ m}$$

$$11. \text{ a) } 0.3420; \sin \frac{\pi}{9} = \sin \left( \frac{\pi}{2} - \frac{7\pi}{18} \right) = \cos \frac{7\pi}{18}$$

$$\text{ b) } 0.3420; \sin \frac{8\pi}{9} = \sin \left( \frac{\pi}{2} + \frac{7\pi}{18} \right) = \cos \frac{7\pi}{18}$$

$$12. \text{ a) } \frac{-\sqrt{3} - 1}{2\sqrt{2}}$$

$$13. \text{ a) } \cos x = -\frac{24}{25} \text{ b) } \sin y = \frac{12}{13} \text{ c) } -\frac{36}{325}$$

14. Yes; the engine's maximum velocity ( $293.2$  rad/s) is slower than the maximum velocity of the propeller ( $300$  rad/s).

15.  $(5000 + 2500\sqrt{3})$  km

18. Answers may vary. Sample answer: Let  $x = 0$  and  $y = \frac{\pi}{2}$ .

$$20. \frac{2.4(\sqrt{3} - 1)}{\sqrt{3}} \text{ m}$$

$$21. \text{ a) } \frac{\pi}{2} \text{ b) } A \left( \frac{\pi}{3}, 0.5 \right), B \left( \frac{5\pi}{3}, 0.5 \right); \cos x = \cos(2\pi - x)$$

$$\text{ c) } \cos x = -\sin \left( \frac{3\pi}{2} - x \right)$$

d) No. An identity must be proven algebraically.

#### CHAPTER 5

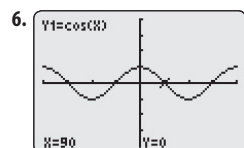
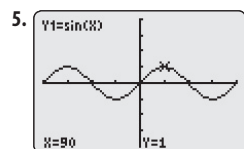
##### Prerequisite Skills, pages 250–251

1. a) 0.5878 b) 0.9659 c)  $-5.6713$  d)  $-0.4142$

2. a) 5.9108 b) 32.4765 c) 0.3773 d)  $-1.4479$

$$3. \text{ a) } -\frac{1}{\sqrt{2}} \text{ b) } \frac{\sqrt{3}}{2} \text{ c) } -1 \text{ d) } \frac{1}{2} \text{ e) } \frac{1}{2} \text{ f) } \sqrt{3}$$

4. a)  $-\frac{2}{\sqrt{3}}$  b)  $-\frac{2}{\sqrt{3}}$  c)  $-1$  d)  $1$  e)  $0$  f)  $\sqrt{2}$



7. The graphs of sine and cosine are periodic because they repeat a pattern of y-values at regular intervals of their domain.

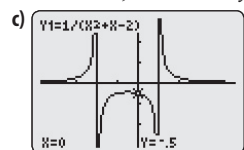
8. a) amplitude 3, period  $180^\circ$ , phase shift  $30^\circ$  to the right, vertical translation 1 unit downward b) maximum 2, minimum  $-4$  c)  $39.7^\circ, 110.3^\circ, 219.7^\circ$  d)  $-3.6$

9. a) amplitude 2, period  $360^\circ$ , phase shift  $90^\circ$  to the left, vertical translation 1 unit upward b) maximum 3, minimum  $-1$  c)  $210^\circ, 330^\circ, 570^\circ$  d)  $1$

10. a)  $31.3^\circ$  b)  $141.3^\circ$  c)  $74.3^\circ$  d)  $27.9^\circ$

11. a)  $0.2$  b)  $2.3$  c)  $0.9$  d)  $0.2$

12. a)  $x = 1, x = -2$  b)  $y = 0$



13. a) 3; the function is linear, so the rate of change is the slope of the line. b) same

14.  $14.4 \text{ km/h}$

15. a)  $17 \text{ m/s}$  b)  $15 \text{ m/s}$  c) the speed at  $0.5 \text{ s}$

### 5.1 Graphs of Sine, Cosine, and Tangent Functions, pages 252–260

1. a) maxima  $(-\frac{3\pi}{2}, 5), (\frac{\pi}{2}, 5);$

minima  $(-\frac{\pi}{2}, 3), (\frac{3\pi}{2}, 3)$

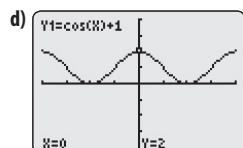
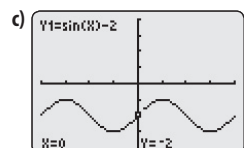
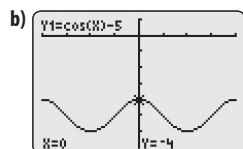
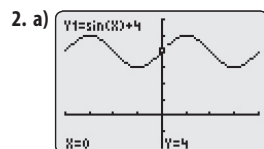
b) maxima  $(-2\pi, -4), (0, -4), (2\pi, -4);$

minima  $(-\pi, -6), (\pi, -6)$

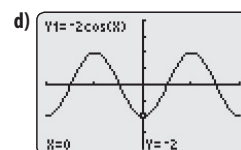
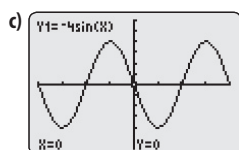
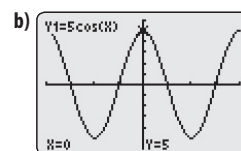
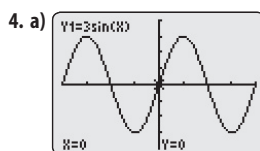
c) maxima  $(-\frac{3\pi}{2}, -1), (\frac{\pi}{2}, -1);$

minima  $(-\frac{\pi}{2}, -3), (\frac{3\pi}{2}, -3)$

d) maxima  $(0, 2), (2\pi, 2);$  minima  $(\pi, 0)$

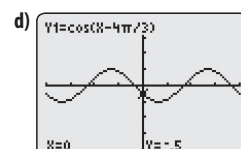
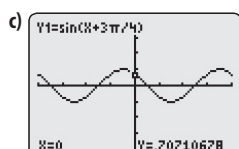
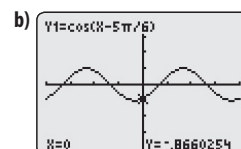
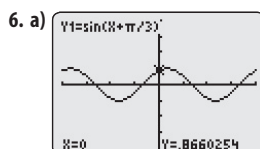


3. a)  $y = 3 \sin x$  b)  $y = 5 \cos x$  c)  $y = -4 \sin x$  d)  $y = -2 \cos x$

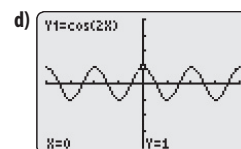
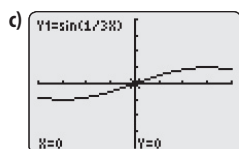
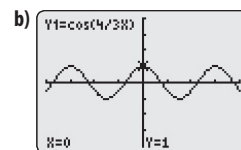
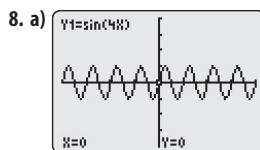


5. a)  $y = \sin(x + \frac{\pi}{3})$  b)  $y = \cos(x - \frac{5\pi}{6})$  c)  $y = \sin(x + \frac{3\pi}{4})$

d)  $y = \cos(x - \frac{4\pi}{3})$

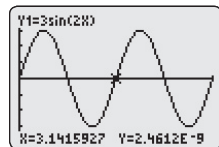


7. a)  $y = \sin 4x$  b)  $y = \cos \frac{4}{3}x$  c)  $y = \sin \frac{1}{3}x$  d)  $y = \cos 2x$



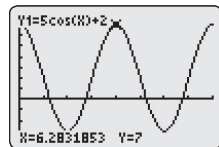
9. a)  $y = 3 \sin 2x$

b) Window variables:  $x \in [0, 2\pi], X\text{scl} \frac{\pi}{2}, y \in [-4, 4]$



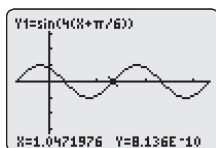
10. a) 5 b) 2 units upward

c) Window variables:  $x \in [0, 4\pi], X\text{scl} \frac{\pi}{2}, y \in [-4, 8]$



11. a)  $\frac{\pi}{2}$  b)  $\frac{\pi}{6}$  rad to the left c)  $y = \sin\left[4\left(x + \frac{\pi}{6}\right)\right]$

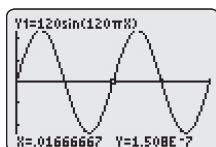
d) Window variables:  $x \in \left[-\frac{\pi}{6}, \frac{5\pi}{6}\right]$ , Xscl  $\frac{\pi}{4}$ ,  $y \in [-4, 4]$



12. a)  $\frac{1}{440}$  b)  $880\pi$

13. a) 120 b)  $\frac{1}{60}$  c)  $y = 120 \sin 120\pi x$

d) Window variables:  $x \in \left[0, \frac{1}{30}\right]$ , Xscl  $\frac{1}{120}$ ,  $y \in [-150, 150]$ , Yscl 50



14. a) Odd. The graph of  $y = \sin(-x)$  is equivalent to the graph of  $y = -\sin x$ .

b) Even. The graph of  $y = \cos(-x)$  is equivalent to the graph of  $y = \cos x$ .

c) Odd. The graph of  $y = \tan(-x)$  is equivalent to the graph of  $y = -\tan x$ .

15. Answers may vary.

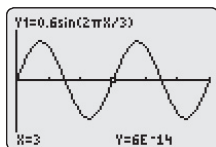
16. g) For positive  $x_A$ , the amplitude gets larger as  $x_A$  gets larger. For negative  $x_A$ , the amplitude gets larger as  $x_A$  gets larger, but the graph of  $y = \sin x$  is reflected in the  $x$ -axis.

h) The amplitude range changes.

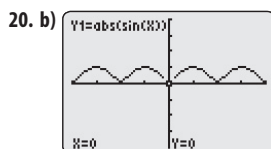
17. a)  $a = \frac{3}{2}$  b)  $c = \frac{5}{2}$  c) The period is 60 s. d)  $k = \frac{\pi}{30}$

18. a)  $d = 0.6 \sin\left(\frac{2\pi}{3}t\right)$

b) Window variables:  $x \in [0, 6]$ ,  $y \in [-1, 1]$



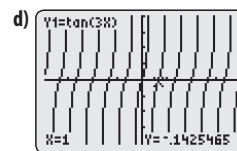
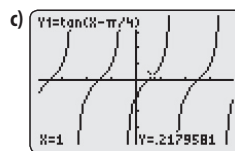
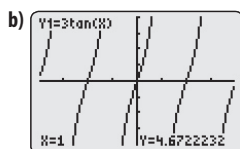
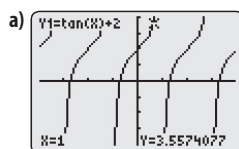
c) The waves will be closer together. The equation becomes  $d = 0.6 \sin \pi t$ .



c) Yes; it passes the vertical line test.

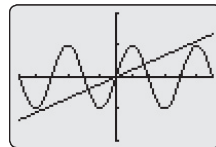
d) Even; it is symmetric about the  $y$ -axis.

21. Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



22. a) 3

b) Window variables:  $x \in [-3\pi, 3\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 2]$



23. a)  $\frac{\pi}{3}$  b)  $\frac{\pi}{6}$  c)  $\frac{\pi}{4}$  d)  $\frac{3\pi}{4}$

24. a) i)  $r^2 = 4$  ii)  $r = \frac{5}{3 \cos \theta + 4 \sin \theta}$  iii)  $r = 4 \sin \theta + 1$

b) i)  $x^2 + y^2 = 36$  ii)  $x^2 + y^2 = 3x$  iii)  $x^2 + y^2 = 2x + 2y$

## 5.2 Graphs of Reciprocal Trigonometric Functions, pages 261–269

1.  $x \doteq 0.20$ ,  $x \doteq 2.94$

2.  $x \doteq 1.05$ ,  $x \doteq 5.24$

3.  $x \doteq 2.90$ ,  $x \doteq 6.04$

4. a) The cosecant function is the reciprocal of the sine function and  $\sin^{-1}$  is the opposite operation of sine.

b)  $\csc \frac{1}{\sqrt{2}} \doteq 1.5393$ ,  $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

5. a) The secant function is the reciprocal function of the cosine function and  $\cos^{-1}$  is the opposite operation of cosine.

b)  $\sec \frac{\sqrt{3}}{2} = 1.5425$ ,  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

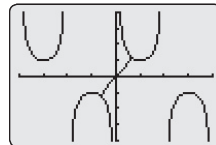
6. a) The cotangent function is the reciprocal of the tangent function and  $\tan^{-1}$  is the opposite operation of tangent.

b)  $\cot 1 \doteq 0.6421$ ,  $\tan^{-1}(1) = \frac{\pi}{4}$

7. a)  $\sec x = \csc\left(x + \frac{\pi}{2}\right)$  or  $\sec x = \csc\left(x - \frac{3\pi}{2}\right)$

b) Answers may vary. Yes; the phase shift can be increased or decreased by one period,  $2\pi$ .

8. a) Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$

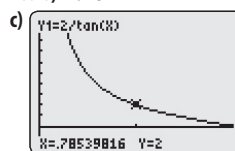


b)  $x \doteq 0.944$  or  $x \doteq -0.944$

9. b) The range is  $0 \leq x \leq \tan^{-1}(2)$  or approximately  $0 \leq x \leq 1.107$ . c) Assuming the lifeguard swims a portion of the distance,  $w \leq d \leq \sqrt{5}w$ .

d) Answers may vary. e) The total distance will be shorter.

10. b) 1.15 m

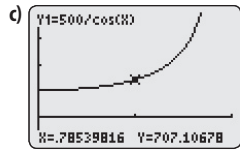


d) As  $x$  approaches 0,  $d$  approaches infinity. This means that the angle of elevation on the summer solstice approaches the horizon and so the length of the awning approaches infinity. As  $x$  approaches  $\frac{\pi}{2}$ ,  $d$  approaches 0. This means that the angle of elevation on the summer solstice approaches an overhead location and the length of the awning approaches 0.

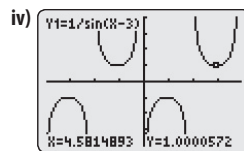
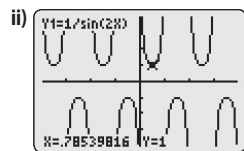
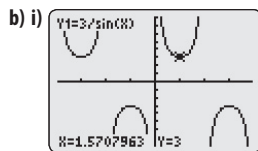
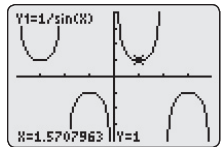
11. a)  $x \approx 0.70$  b) No;  $x \approx 0.40$ . c) No;  $x \approx 1.28$ .

12. a) Answers may vary. Sample answer:  $\csc^2 x - 1 = \cot^2 x$ .

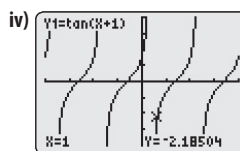
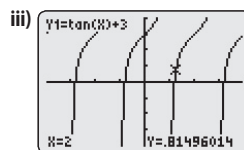
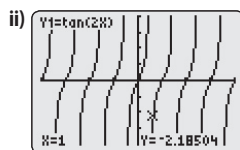
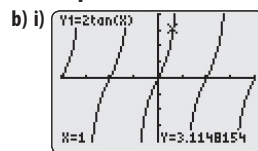
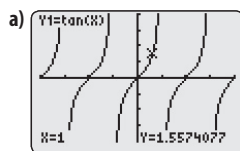
13. a)  $d = 500 \sec x$  b)  $d = \frac{1000\sqrt{2}}{\sqrt{3} - 1}$



15. a) Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



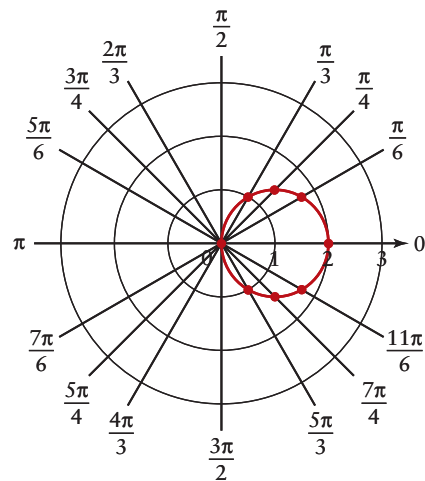
16. Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



17. b) No; the equation is only true for  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

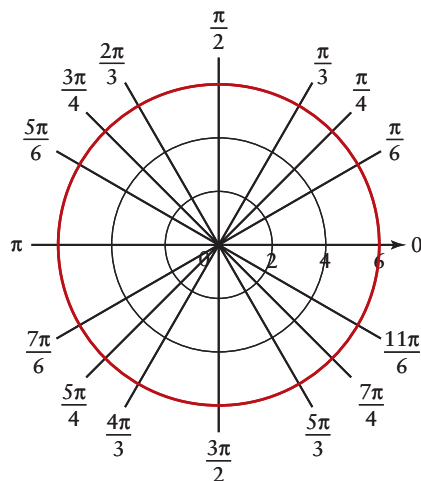
18.  $\frac{3}{5}$

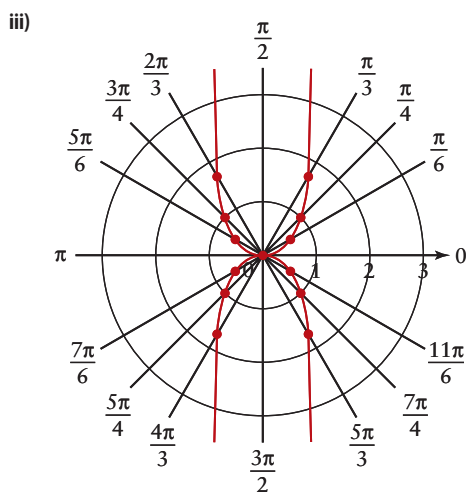
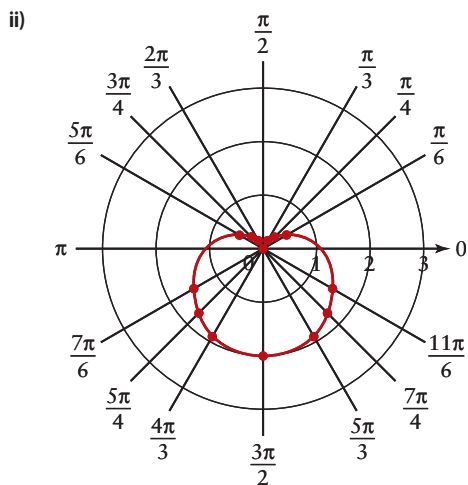
19. a)



$\theta$	$r = 2 \cos \theta$	$(r, \theta)$
0	2	(2, 0)
$\frac{\pi}{6}$	$\sqrt{3}$	$(\sqrt{3}, \frac{\pi}{6})$
$\frac{\pi}{4}$	$\frac{2}{\sqrt{2}}$	$(\frac{2}{\sqrt{2}}, \frac{\pi}{4})$
$\frac{\pi}{3}$	1	$(1, \frac{\pi}{3})$
$\frac{\pi}{2}$	0	$(0, \frac{\pi}{2})$
$\frac{2\pi}{3}$	-1	$(-1, \frac{2\pi}{3})$
$\frac{3\pi}{4}$	$-\frac{2}{\sqrt{2}}$	$(-\frac{2}{\sqrt{2}}, \frac{3\pi}{4})$
$\frac{5\pi}{6}$	$-\sqrt{3}$	$(-\sqrt{3}, \frac{5\pi}{6})$
$\pi$	-2	$(-2, \pi)$

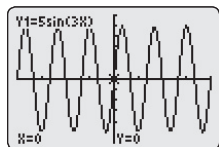
b) i)



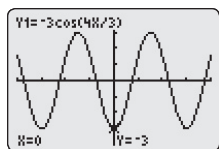


### 5.3 Sinusoidal Functions of the Form $f(x) = a \sin[k(x - d)] + c$ and $f(x) = a \cos[k(x - d)] + c$ , pages 270–279

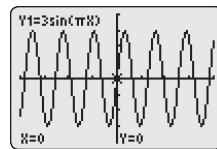
1. a) amplitude 5, period  $\frac{2\pi}{3}$   
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-6, 6]$



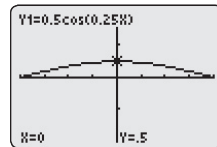
b) amplitude 3, period  $\frac{3\pi}{2}$   
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



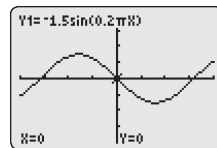
c) amplitude 3, period 2  
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



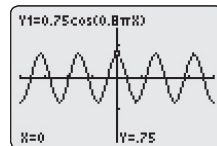
d) amplitude  $\frac{1}{2}$ , period 8π  
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 2]$



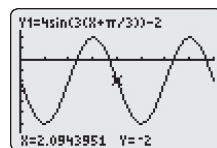
e) amplitude 1.5, period 10  
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



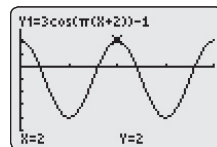
f) amplitude 0.75, period 2.5  
Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 2]$



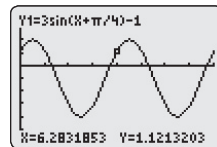
2. a)  $y = 3 \sin 4x$  b)  $y = \frac{1}{2} \cos 2\pi x$   
3. a) 4 b)  $\frac{2\pi}{3}$  c)  $\frac{\pi}{3}$  rad to the left d) 2 units downward  
e) Window variables:  $x \in [0, \frac{4\pi}{3}]$ , Xscl  $\frac{\pi}{6}$ ,  $y \in [-8, 4]$



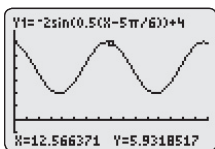
4. a) 3 b) 2 c) 2 rad to the left d) 1 unit downward  
e) Window variables:  $x \in [0, 4]$ ,  $y \in [-6, 4]$



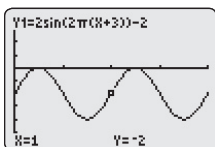
5. a) amplitude 3, period  $2\pi$ , phase shift  $\frac{\pi}{4}$  rad to the left, vertical translation 1 unit downward  
Window variables:  $x \in [0, 4\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-6, 4]$



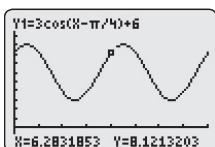
b) amplitude 2, period  $4\pi$ , phase shift  $\frac{5\pi}{6}$  rad to the right, vertical translation 4 units upward  
Window variables:  $x \in [0, 8\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 8]$



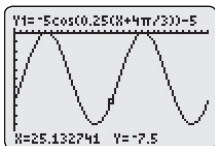
c) amplitude 2, period 1, phase shift 3 rad to the left, vertical translation 2 units downward  
Window variables:  $x \in [0, 2]$ , Xscl 0.5,  $y \in [-6, 4]$



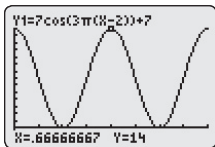
6. a) amplitude 3, period  $2\pi$ , phase shift  $\frac{\pi}{4}$  rad to the right, vertical translation 6 units upward  
Window variables:  $x \in [0, 4\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 12]$



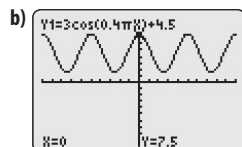
b) amplitude 5, period  $8\pi$ , phase shift  $\frac{4\pi}{3}$  rad to the left, vertical translation 5 units downward  
Window variables:  $x \in [0, 16\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-12, 2]$



c) amplitude 7, period  $\frac{2}{3}$ , phase shift 2 rad to the right, vertical translation 7 units upward  
Window variables:  $x \in [0, \frac{4}{3}]$ , Xscl  $\frac{1}{6}$ ,  $y \in [-2, 16]$

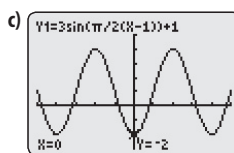


7. a)  $h = 3 \cos(0.4\pi t) + 4.5$



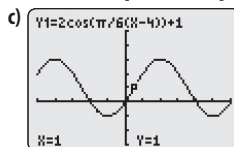
8. a) amplitude 3, period 4, phase shift 1 rad to the right, vertical translation 1 unit upward

b)  $y = 3 \sin\left[\frac{\pi}{2}(x - 1)\right] + 1$



9. a) amplitude 2, period 12, phase shift 4 rad to the right, vertical translation 1 unit upward

b)  $y = 2 \cos\left[\frac{\pi}{6}(x - 4)\right] + 1$

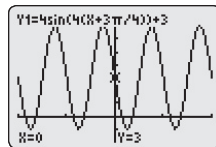


10. a)  $y = 3 \sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 1$  b)  $y = 2 \sin\left[\frac{\pi}{3}(x + 2)\right] + 2$

11. a)  $y = 4 \cos\left[1.5\left(x + \frac{\pi}{3}\right)\right] + 1$  b)  $y = 2.5 \cos\left[\frac{\pi}{4}(x - 2)\right] - 1.5$

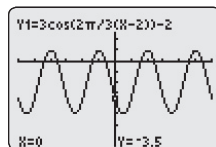
12. a)  $y = 4 \sin\left[4\left(x + \frac{3\pi}{4}\right)\right] + 3$

b) Window variables:  $x \in [-\pi, \pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-2, 8]$



13. a)  $y = 3 \cos\left[\frac{2\pi}{3}(x - 2)\right] - 2$

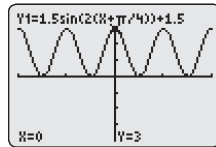
b) Window variables:  $x \in [-6, 6]$ ,  $y \in [-8, 4]$



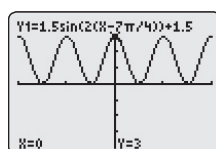
14. Answers may vary.

a)  $y = 1.5 \sin\left[2\left(x + \frac{\pi}{4}\right)\right] + 1.5$

b) Window variables:  $x \in [-2\pi, 2\pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



c) Yes.  $y = 1.5 \sin\left[2\left(x - \frac{7\pi}{4}\right)\right] + 1.5$



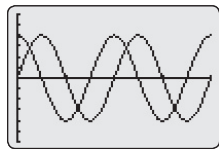
15. a)  $2 \cos\left(\frac{4\pi}{3}x\right)$ ,  $b) \left(0, 1\right), \left(\frac{4\pi}{3}, -\frac{1}{2}\right)$

16. Answers may vary.

17. Answers may vary.

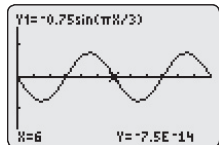
18. a)  $x = 4 \cos \pi t$  b)  $y = 4 \sin \pi t$

c) Window variables:  $x \in [0, 4], y \in [-6, 6]$



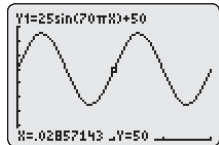
19. a)  $v = -\frac{3}{4} \sin \frac{\pi}{3} x$

b) Window variables:  $x \in [0, 12], y \in [-2, 2]$



20. a)  $h = 25 \sin(70\pi t) + 50$

b) Window variables:  $x \in \left[0, \frac{2}{35}\right], Xscl \frac{1}{70}, y \in [0, 90]$



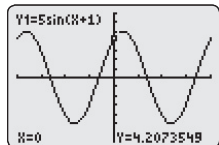
c) Only the period changes:  $h = 25 \sin(80\pi t) + 50$ .

22.  $y = a \csc[k(x - d)] + c$ ;  $a$ : multiply  $y$ -value by  $a$ ;

$k$ : changes the period to  $\frac{2\pi}{k}$ ;  $d$ : phase shifts work the same as for sinusoidal functions;  $c$ : vertical translations work the same as for sinusoidal functions.

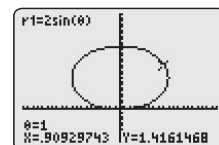
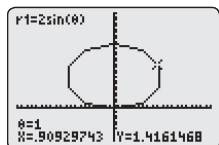
23. a), b) 2.5 s to the right

24. a) Window variables:  $x \in [-2\pi, 2\pi], Xscl \frac{\pi}{2}, y \in [-7, 7]$



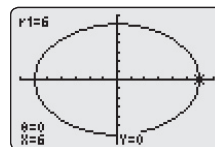
c)  $a = 2.7$  and  $b = 4.2$ , to two decimal places.

25. a)

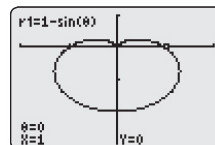


Smaller increments of  $\theta$  step make the graph smoother (more circular).

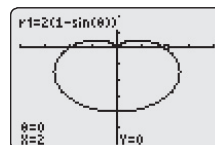
b) i) Window variables:  $\theta \in [0, 2\pi], \theta$  step  $\frac{\pi}{12}, x \in [-7, 7], y \in [-7, 7]$



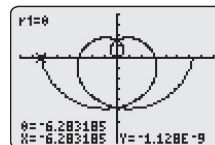
ii) Window variables:  $\theta \in [0, 2\pi], \theta$  step  $\frac{\pi}{100}, x \in [-2, 2], y \in [-3, 1]$



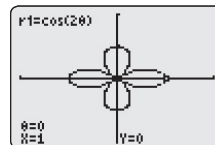
iii) Window variables:  $\theta \in [0, 2\pi], \theta$  step  $\frac{\pi}{100}, x \in [-4, 4], y \in [-6, 2]$



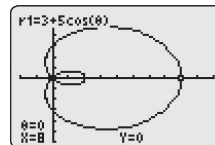
iv) Window variables:  $\theta \in [-2\pi, 2\pi], \theta$  step  $\frac{\pi}{100}, x \in [-8, 8], y \in [-8, 4]$



v) Window variables:  $\theta \in [0, 2\pi], \theta$  step  $\frac{\pi}{100}, x \in [-2, 2], y \in [-2, 2]$



iv) Window variables:  $\theta \in [0, 2\pi], \theta$  step  $\frac{\pi}{100}, x \in [-2, 10], y \in [-6, 6]$



## Extension, page 280

### Part 1

1. a)

$x$	$y$
0	0
$0.5\pi$	1
$\pi$	0
$1.5\pi$	-1
$2\pi$	0

4. The  $k$ -value has not been factored out of the bracket.

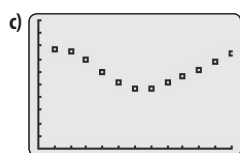
6. Answers may vary.



Part 2

1. a)

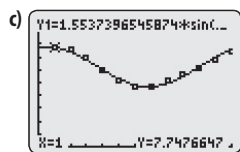
Sunrise in Fort Erie, ON		
Date	Time	Time (decimals)
Jan 1	7:47	7.78
Feb 1	7:31	7.52
Mar 1	6:52	6.87
Apr 1	5:58	5.97
May 1	5:10	5.17
Jun 1	4:40	4.67
Jul 1	4:40	4.67
Aug 1	5:06	5.10
Sep 1	5:40	5.67
Oct 1	6:12	6.20
Nov 1	6:49	6.82
Dec 1	7:26	7.43



Window variables:  $x \in [0, 12]$ ,  $y \in [0, 10]$

2. a)  $y = 1.553\ 739\ 6 \sin(0.497\ 842\ 4x + 1.304\ 949\ 9) + 6.235\ 505\ 6$

b) amplitude 1.6, maximum 7.78, minimum 4.67, period  $4\pi$ , phase shift 2.6, vertical translation up 1.3



4. Answers may vary.

5.4 Solve Trigonometric Equations, pages 282–289

1. a) 0.25, 2.89 b) 2.42, 3.86 c) 1.37, 4.51 d) 1.32, 4.97

e) 2.16, 5.30 f) 3.55, 5.87

3. a)  $\frac{4\pi}{3}, \frac{5\pi}{3}$  b)  $\frac{\pi}{3}, \frac{5\pi}{3}$  c)  $\frac{\pi}{4}, \frac{5\pi}{4}$  d)  $\frac{3\pi}{4}, \frac{7\pi}{4}$

5. a) 0.93, 2.21, 4.07, 5.36 b) 0.84, 2.30, 3.98, 5.44

c) 0.88, 2.27, 4.02, 5.41 d) 0.89, 2.26, 4.03, 5.40

e) 0.74, 2.40, 3.88, 5.55

7. a)  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$  b)  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$  c)  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

d)  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

9.  $\frac{3\pi}{2}$

10.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

11.  $\pi$

12. 1.11, 1.89, 4.25, 5.03

13. a) 0.46, 1.11 b) 0.32, 1.25 c) 0.42, 1.15

14. no solution

15. 0.17, 1.40, 3.31, 4.54

16. 0.84, 5.44

17.  $\frac{\pi}{6}, \frac{5\pi}{6}$

18. 1.91, 4.37,  $\frac{2\pi}{3}, \frac{4\pi}{3}$

19. 1.25, 2.68, 4.39, 5.82

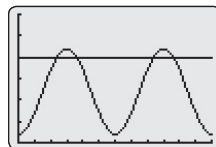
20. a) No two integers have a product of  $-3$  and a sum of 1.

b)  $\frac{-1 \pm \sqrt{13}}{6}$  c) 0.45, 2.69, 4.02, 5.41

21. b) Technology allows you to check all the zeros on the graph within the domain.

22. a) 11.93 s, 18.07 s

b) Window variables:  $x \in [0, 60]$ , Xscl 5,  $y \in [0, 30]$ , Yscl 5



23. 0.08

24. 0.53, 1.04

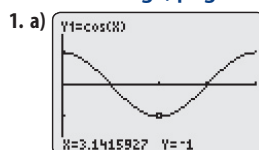
26.  $-2\pi, -\pi, 0, \pi, 2\pi$

27. a) 0.004 s b) No.

28.  $-2\pi, 0, 2\pi$

29. Answers may vary.

5.5 Making Connections and Instantaneous Rate of Change, pages 290–299

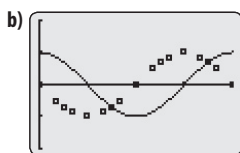


b) 0,  $\pi$ ,  $2\pi$

c) maximum  $\frac{3\pi}{2}$ , minimum  $\frac{\pi}{2}$

2. a)

Angle $x$	$f(x) = \cos x$	Instantaneous Rate of Change
0	1	0
$\frac{\pi}{6}$	0.87	-0.50
$\frac{\pi}{4}$	0.71	-0.71
$\frac{\pi}{3}$	0.50	-0.87
$\frac{\pi}{2}$	0	-1
$\frac{2\pi}{3}$	-0.50	-0.87
$\frac{3\pi}{4}$	-0.71	-0.71
$\frac{5\pi}{6}$	-0.87	-0.50
$\pi$	-1	0
$\frac{7\pi}{6}$	-0.87	0.50
$\frac{5\pi}{4}$	-0.71	0.71
$\frac{4\pi}{3}$	-0.50	0.87
$\frac{3\pi}{2}$	0	1
$\frac{5\pi}{3}$	0.50	0.87
$\frac{7\pi}{4}$	0.71	0.71
$\frac{11\pi}{6}$	0.87	0.50
$2\pi$	1	0



c) Yes.

3. a) i)  $-0.174$  ii)  $-0.192$  iii)  $-0.196$  iv)  $-0.196$

b) The instantaneous rate of change of  $b$  at  $t = 20$  s is about  $-0.196$  m/s.

c) The instantaneous rate of change represents the vertical speed of the car at  $t = 20$  s.

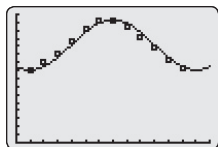
d) No. The graph of the sine function changes its slope continually and would not likely yield the same value at a different value of  $t$ .

4. a)

Daylight in Sarnia, ON	
Month	Duration (decimal)
1	9.08
2	9.95
3	11.20
4	12.73
5	14.10
6	15.13
7	15.32
8	14.52
9	13.18
10	11.75
11	10.30
12	9.25

b)  $T = 3.12 \sin\left[\frac{\pi}{6}(m - 5)\right] + 12.2$

c) Window variables:  $x \in [0, 14], y \in [0, 16]$



The equation fits the data well.

d)  $T \doteq 3.11 \sin[0.51(m - 3.63)] + 12.14$

The values for  $a, k, c,$  and  $d$  compare well with those in the model.

e) phase shift

f)  $T \doteq 3.11 \cos[0.51(m - 6.71)] + 12.14$

5. 1.5 h/month

6.  $b = 3 \cos(\pi t) + 4$

7. a) (1.5, 4) b) 9.4 m/s c) speed of the spring

8. Answers may vary.

9. Answers may vary.

10. a)  $a = 4, k = 5\pi$  b)  $d = 4 \sin 5\pi t$

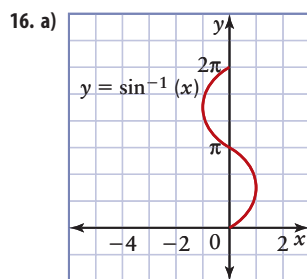
11. a)  $y = -x^2 + 8$  b)  $y = 2 \sin\left(\frac{\pi}{2}x\right) + 4$

12. a)  $-4$  b) 3.14 c) different

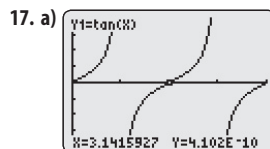
d) Answers may vary. Sample answer: The cars may fall off the track.

13. Answers may vary.

14. Answers may vary.



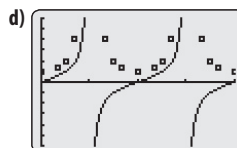
b) No. Restrict the range to the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ . c) 0 d) 1



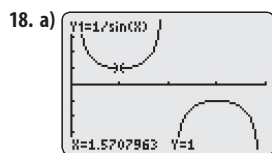
b) none; no maximum; no minimum

c)

Angle $x$	$f(x) = \tan x$	Instantaneous Rate of Change
0	0	1
$\frac{\pi}{6}$	0.58	1.33
$\frac{\pi}{4}$	1	2
$\frac{\pi}{3}$	1.73	4
$\frac{\pi}{2}$	undefined	undefined
$\frac{2\pi}{3}$	-1.73	4
$\frac{3\pi}{4}$	-1	2
$\frac{5\pi}{6}$	-0.58	1.33
$\pi$	0	1
$\frac{7\pi}{6}$	0.58	1.33
$\frac{5\pi}{4}$	1	2
$\frac{4\pi}{3}$	1.73	4
$\frac{3\pi}{2}$	undefined	undefined
$\frac{5\pi}{3}$	-1.73	4
$\frac{7\pi}{4}$	-1	2
$\frac{11\pi}{6}$	-0.58	1.33
$2\pi$	0	1



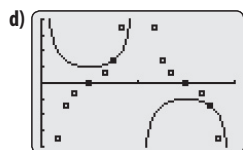
e) Answers may vary.



b)  $\frac{\pi}{2}, \frac{3\pi}{2}$ ; no maximum; no minimum

c)

Angle $x$	$f(x) = \csc x$	Instantaneous Rate of Change
0	undefined	undefined
$\frac{\pi}{6}$	2	-3.46
$\frac{\pi}{4}$	1.41	-1.41
$\frac{\pi}{3}$	1.15	-0.67
$\frac{\pi}{2}$	1	0
$\frac{2\pi}{3}$	1.15	0.67
$\frac{3\pi}{4}$	1.41	1.41
$\frac{5\pi}{6}$	2	3.46
$\pi$	undefined	undefined
$\frac{7\pi}{6}$	-2	3.46
$\frac{5\pi}{4}$	-1.41	1.41
$\frac{4\pi}{3}$	-1.15	0.67
$\frac{3\pi}{2}$	-1	0
$\frac{5\pi}{3}$	-1.15	-0.67
$\frac{7\pi}{4}$	-1.41	-1.41
$\frac{11\pi}{6}$	-2	-3.46
$2\pi$	undefined	undefined



e) Answers may vary.

19. 1

20. a)  $\frac{1}{2}$  b)  $\angle R = \frac{\pi}{6}$

21.  $r = a \sin \theta + b \cos \theta$  is a circle

$$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

with center  $\left(\frac{b}{2}, \frac{a}{2}\right)$  and radius  $\frac{\sqrt{a^2 + b^2}}{2}$ .

### Chapter 5 Review, pages 300–301

1. a) 2 b) 4

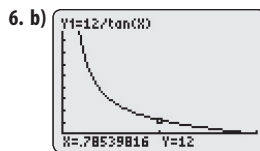
2.  $y = \cos\left[\frac{2}{3}\left(x + \frac{\pi}{3}\right)\right]$

3. a)  $y = 5 \sin 60\pi t$  b) No. A phase shift can generate another possible equation.

4. 0.25, 2.89

5. a) The secant function is a reciprocal of the cosine function and  $\cos^{-1}$  is the opposite operation of cosine.

b)  $\sec\left(\frac{1}{\sqrt{2}}\right) \doteq 1.32, \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

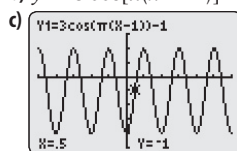


c) As  $x$  approaches 0,  $s$  approaches infinity. This means that the angle of elevation of the Sun approaches the horizon and so the length of the shadow approaches infinity.

As  $x$  approaches  $\frac{\pi}{2}$ ,  $s$  approaches 0. This means that the angle of elevation of the Sun approaches an overhead location and the length of the shadow approaches 0.

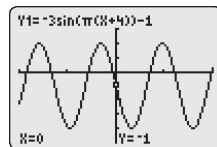
7. a) amplitude 3, vertical translation 1 unit downward, phase shift 1 rad to the right, period 2

b)  $y = 3 \cos[\pi(x - 1)] - 1$



8. a) 3 b) 2 c) 4 rad to the left d) 1 unit downward

e) Window variables:  $x \in [-\pi, \pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-5, 4]$



9. a) 1.32, 4.97 b) 0.64, 2.50 c) 0.46, 3.61 d) no solution

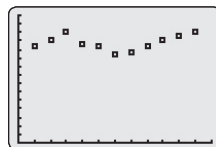
10. a)  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

11.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12. a)  $y = 0.9 \sin\left(\frac{\pi}{2}t\right)$  b) 0.37 s, 1.63 s

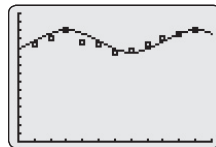
13. 1 s, 3 s; maxima 0 s, 4 s

14. a) Window variables:  $x \in [0, 12]$ ,  $y \in [0, 300]$ , Yscl 20



b)  $y = 26 \sin\left[\frac{\pi}{4}(x - 1)\right] + 235$

c) The equation fits the data reasonably well.



d) Using sinusoidal regression, an equation that better fits the data is  $y \doteq 22.68 \sin[0.83(x - 0.96)] + 230.61$ .

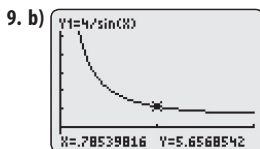
e) Answers may vary.

Chapter 5 Practice Test, pages 302–303

1. B
2. C
3. C
4. A
5. D
6. C
7. A

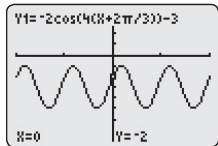
8. a) The cosecant function is a reciprocal of the sine function and  $\sin^{-1}$  is the opposite operation of sine.

b)  $\csc\left(\frac{\sqrt{3}}{2}\right) \doteq 1.31, \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$



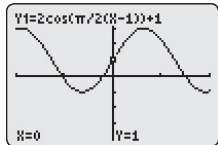
c) As  $x$  approaches 0,  $l$  approaches infinity. This means that the angle of inclination of the wire approaches horizontal and so the length of the wire approaches infinity. As  $x$  approaches  $\frac{\pi}{2}$ ,  $l$  approaches 4. This means that the angle of inclination of the wire approaches vertical and the length of the wire approaches 4 m.

10. a) 2 b)  $\frac{\pi}{2}$  c)  $\frac{2\pi}{3}$  rad to the left d) 3 units downward  
e) Window variables:  $x \in [-\pi, \pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-8, 4]$



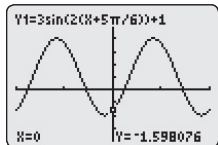
11. a)  $y = 2 \cos\left[\frac{\pi}{2}(x - 1)\right] + 1$

- b) Window variables:  $x \in [-\pi, \pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



12. a)  $y = 3 \sin\left[2\left(x + \frac{5\pi}{6}\right)\right] + 1$

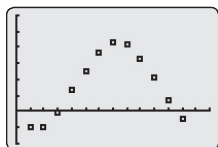
- b) Window variables:  $x \in [-\pi, \pi]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 6]$



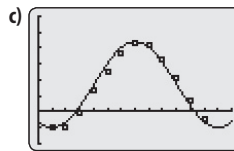
13. 0.80, 2.35, 3.88, 5.49

14.  $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

15. a) Window variables:  $x \in [0, 14]$ ,  $y \in [-10, 30]$ , Xscl 5



b)  $y = 13.4 \sin\left[\frac{\pi}{6}(x - 4)\right] + 8.55$



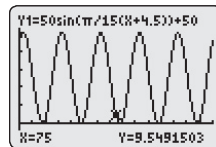
The model appears to fit the data well.

16. a), b)

Phases of the Moon 2007	
Date (days from beginning of year)	Phase (percent illumination)
3	100
11	50
19	0
25	50
33	100
41	50
48	0
55	50
62	100
71	50
78	0
84	50
92	100
100	50
107	0
114	50

c)  $y = 50 \sin\left[\frac{\pi}{15}(x + 4.5)\right] + 50$

- d) Window variables:  $x \in [0, 150]$ , Xscl 10,  $y \in [-20, 120]$ , Yscl 20



e)  $y \doteq 49.75 \sin[0.21(x + 4)] + 52.74$ ; the values for  $a$ ,  $k$ ,  $c$ , and  $d$  compare well with those in the model.

17. a) Answers may vary. Sample Answer: Using your model, first find the average rate of change of the percent of illumination, and then estimate the instantaneous rate of change.

b) The instantaneous rate of change on January 25 is about 10.4%/day.

c) The instantaneous rate of change represents the percent change in illumination of the Moon per day.

Chapters 4 and 5 Review, pages 304–305

1. a)  $\frac{5\pi}{9}$  b)  $105^\circ$

2. 3 radians

3.  $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{6}}$

4.  $25(\sqrt{3} - 1)$  m

5. 0.1045

6.  $\frac{\pi}{8}$

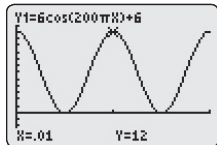
7.  $-\frac{33}{65}$

9. a)  $\frac{\sin^2 x}{\cot^2 x} + \sin^2 x = \tan^2 x$

12. a)  $\frac{7\pi}{12}$  b)  $\frac{\pi}{4}$  to the left

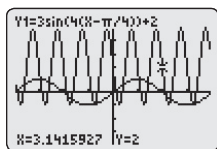
13. a) 6 cm b)  $\frac{1}{100}$  s c)  $200\pi$  d)  $y = 6 \cos(200\pi x) + 6$

e) Window variables:  $x \in [0, 0.02]$ , Xscl 0.01,  $y \in [-4, 15]$



14. a) 3 b)  $\frac{\pi}{2}$  c)  $\frac{\pi}{4}$  to the right d) 2 units upward

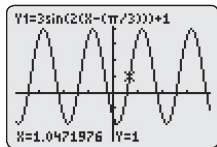
e) Window variables:  $x \in [-\frac{47\pi}{24}, \frac{47\pi}{24}]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 6]$



15. b)  $0 \leq d \leq 250\sqrt{13}$  c) The total time will be a minimum when the contestant stays on the pavement.

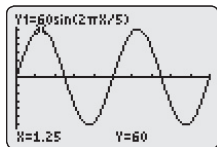
16. a)  $\frac{\pi}{3}$  to the right

b) Window variables:  $x \in [-\frac{47\pi}{24}, \frac{47\pi}{24}]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-3, 5]$



17. a)  $h(t) = 60 \sin(\frac{2\pi t}{5})$

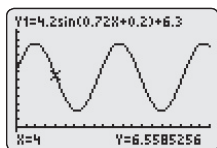
b) Window variables:  $x \in [0, 10]$ ,  $y \in [-80, 80]$ , Yscl 10



c) The value of  $k$  would change from  $\frac{2\pi}{5}$  to  $\frac{2\pi}{3}$ , making the equation  $h(t) = 60 \sin(\frac{2\pi t}{3})$ .

18. a) 1.37, 4.91 b) 0.34, 2.80, 3.39, 6.03

19. a) Window variables:  $x \in [0, 20]$ ,  $y \in [-2, 14]$



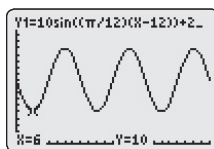
b) 7134 cones c) 6300 cones

d) 8.7 years e) Answers may vary. Sample answer: approximately Oct 1991 and June 2000

f) Answers may vary.

20. a)  $C(t) = 10 \sin[\frac{\pi}{12}(t - 12)] + 20$

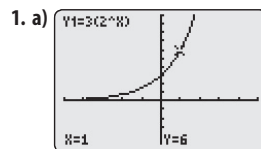
b) Window variables:  $x \in [0, 72]$ , Xscl 3,  $y \in [0, 40]$ , Yscl 2



21. a)  $t = 12$  b) 2.6 ppm/h

## CHAPTER 6

### Prerequisite Skills, pages 308–309



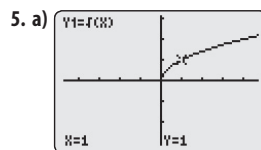
b) i)  $\{x \in \mathbb{R}\}$  ii)  $\{y \in \mathbb{R}, y > 0\}$  iii)  $y = 0$

2. a) 300 b) 2400 c) i) approximately 2.74 days

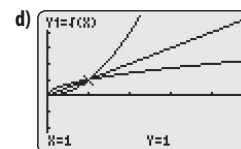
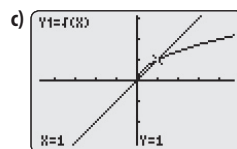
ii) approximately 38 bacteria

3. a)  $x^7$  b)  $m^3$  c)  $k^6$  d)  $-8x^{12}$  e)  $-\frac{2b}{a}$  f)  $\frac{2}{x^2}$  g)  $\frac{1}{u}$

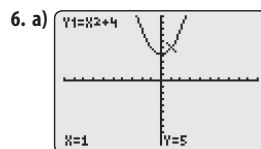
4. a) 4 b) 10 c) 27 d) 82



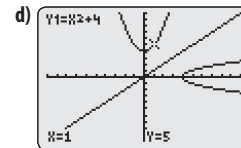
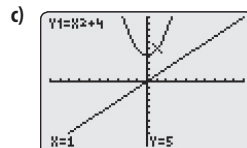
b)  $\{x \in \mathbb{R}, x \geq 0\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$



e) Yes. f)  $\{x \in \mathbb{R}, x \geq 0\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$



b)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 4\}$



e) No. f)  $\{x \in \mathbb{R}, x \geq 4\}$ ,  $\{y \in \mathbb{R}\}$

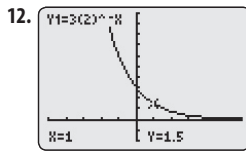
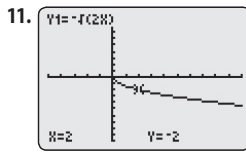
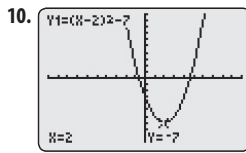
7. Yes. Each curve is a reflection of the other in the line  $y = x$ .

8. a) translation of 3 units to the right and 1 unit up.

b) reflection in the  $x$ -axis, vertical stretch of factor 2

9. a) vertical stretch of factor 3 and then translated down 4 units

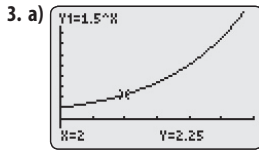
b) period doubled, reflection in the  $y$ -axis



**6.1 The Exponential Function and Its Inverse, pages 318–322**

1. C, D

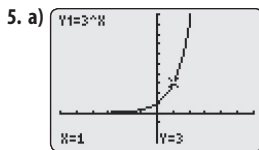
2. C:  $y = 3^x$ ; D:  $y = \left(\frac{1}{3}\right)^x$



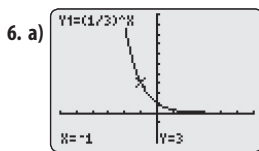
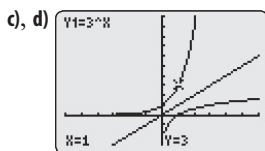
- b) i)  $m_{12} = 0.75$  ii)  $m_{23} = 1.13$  iii)  $m_{34} = 1.69$  iv)  $m_{45} = 2.53$   
 c)  $m_1 = 0.6, m_2 = 0.9, m_3 = 3.4, m_4 = 5.1, m_5 = 7.6$   
 d) increasing



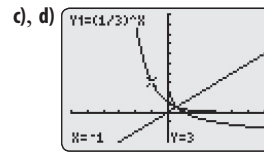
- b) i)  $m_{-3-2} = -4$  ii)  $m_{-2-1} = -2$  iii)  $m_{-10} = -1$  iv)  $m_{01} = -0.5$   
 c)  $m_{-3} = -5.6, m_{-2} = -2.8, m_{-1} = -1.4, m_0 = -0.7, m_1 = -0.35$   
 d) increasing



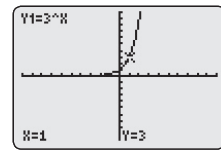
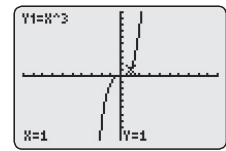
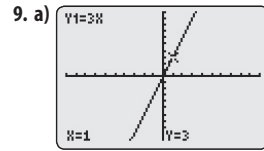
b)  $y = 3^x$



b)  $y = \left(\frac{1}{3}\right)^x$



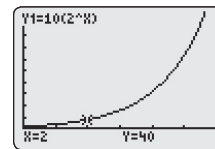
7. a) iii) b) i) c) ii) d) iv)  
 8. a) is inverse of 7d):  $y = \left(\frac{1}{5}\right)^x$   
 b) is inverse of 7b):  $y = 5^x$   
 c) is inverse of 7c):  $y = \left(\frac{1}{2}\right)^x$   
 d) is inverse of 7a):  $y = 2^x$



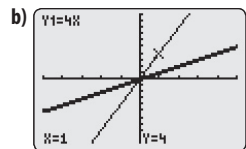
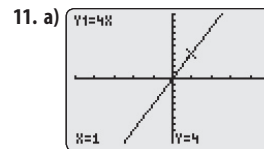
b)

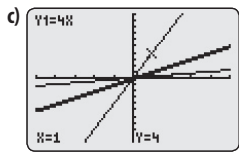
Function	$f(x) = 3x$	$g(x) = x^3$	$h(x) = 3^x$
domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
range	$y \in \mathbb{R}$	$y \in \mathbb{R}$	$y \in \mathbb{R}, y > 0$
x-intercept	(0, 0)	(0, 0)	none
y-intercept	(0, 0)	(0, 0)	(0, 1)
function is negative	$x < 0$	$x < 0$	never
function is positive	$x > 0$	$x > 0$	$x \in \mathbb{R}$
function is increasing	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
equation of asymptote	none	none	$y = 0$

- c) All three functions have the same domain and are increasing.  
 d)  $h(x) = 3^x$  is different than the other two functions for range, x-intercept, y-intercept,  $y = 0$  asymptote, and positive/negative intervals.  
 e) For  $f(x) = 3x$ , the instantaneous rate of change is constant. For  $g(x) = x^3$ , the instantaneous rate of change is decreasing and then increasing. For  $h(x) = 3^x$ , the instantaneous rate of change is increasing.  
 10. a) i) 10 ii) 20 iii) 40 iv) 80 b) Yes.



- c)  $m_{12} = 20$  d) i)  $m_1 = 13.9$  ii)  $m_2 = 27.8$  e) Answers may vary.



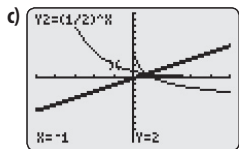
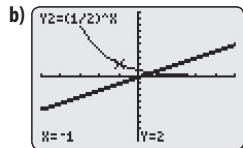
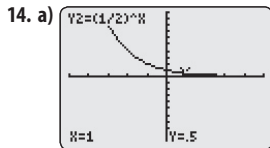


12. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$  b) 0 c) 0

d) For  $x < 0$ , the function is negative. For  $x > 0$ , the function is positive. e) The function is increasing for all intervals. f) None.

13. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$  b) 0 c) 0

d) For  $x < 0$ , the function is negative. For  $x > 0$ , the function is positive. e) The function is increasing for all intervals. f) None.



15. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y > 0\}$  b) None. c) 1

d) The function is positive for all intervals.

e) The function is decreasing for all intervals.

f)  $y = 0$

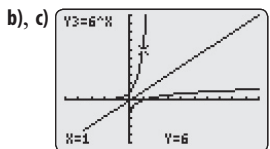
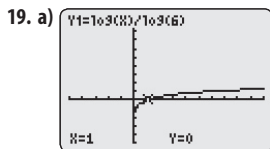
16. a)  $\{x \in \mathbb{R}, x > 0\}, \{y \in \mathbb{R}\}$  b) 1 c) None

d) For  $0 < x < 1$ , the function is positive. For  $x > 1$ , the function is negative.

e) The function is decreasing for  $x > 0$  f)  $x = 0$

17. a) same domain b) the rest of the key features differ

18. a) same range b) the rest of the key features differ

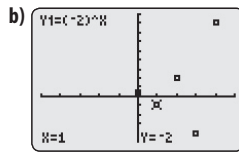


d)  $y = 6$

20.  $y = \left(\frac{1}{10}\right)^x$

21. a)

x	y
0	1
1	-2
2	4
3	-8
4	16



c) No. d) i) not real (square root of a negative value) ii) not real

e) Answers may vary.

22. a)  $m_{12} \doteq 0.56$  km/s,  $m_{34} \doteq 1.10$  km/s b)  $m_3 \doteq 0.9$ ,  $m_4 \doteq 1.3$

23. a) For  $0 < b < 1$ ,  $f$  and  $f^{-1}$  have equal  $x$ - and  $y$ -coordinates at the point where they intersect the line  $y = x$ .

b) Yes; when  $b > 1$ , the graphs do not intersect the line  $y = x$ .

24. c) i)  $b > 0$  ii)  $b < 0$  d) i)  $b > 0$  ii)  $b < 0$  e) Yes.

## 6.2 Logarithms, pages 328–330

1. a)  $\log_4 64 = 3$  b)  $\log_2 128 = 7$  c)  $\log_5 \left(\frac{1}{25}\right) = -2$

d)  $\log_{\frac{1}{2}} 0.25$  e)  $\log_b y = x$  f)  $\log_{10} y = x$  g)  $\log_3 \left(\frac{1}{27}\right) y = -3$

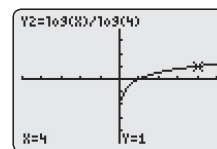
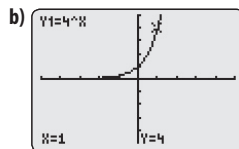
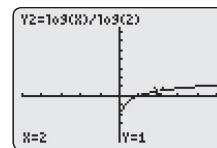
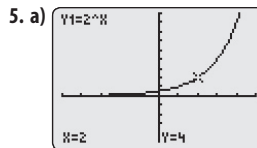
h)  $\log_3 v = n$

2. a) 6 b) 3 c) -2 d) -3 e) 3 f) 10 g) 6 h) 4

3. a) 3 b) -1 c) 0 d) -3 e) -4 f) 6 g) -2 h) 4

4. a)  $7^2 = 49$  b)  $2^5 = 32$  c)  $10^4 = 10\,000$  d)  $b^w = z$  e)  $2^3 = 8$

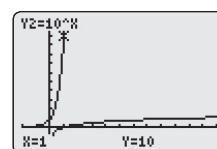
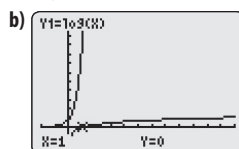
f)  $5^4 = 625$  g)  $10^{-2} = \frac{1}{100}$  h)  $7^{2y} = x$



6. a) 2.6 b) 3.7 c) 6.2 d) -1.5

8. a) 2.63 b) -4.43 c) 0.95 d) -0.70 e) 1.23 f) 2.00 g) 2.26 h) 3.00

9. a)  $y = 10^x$



10. a) 1 b) 1 c) 1 d) 1

11. a)  $\log_x x = 1$  for  $x > 1$ . b) Answers may vary.

Sample answer:  $\log_{11} 11 = 1$  c)  $\log_x x = 1$

12. a) Answers may vary. Sample answer: The logarithmic function has decreasing slope and the exponential function has increasing slope. b) Answers may vary.

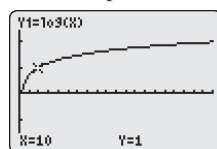
13. a) approximately 6.3 days b) approximately 12.6 days

14. a) approximately 66.5 m b) No.  $d \doteq 16.2$  m.

c) Answers may vary. Sample answer: Drive slower.

15. a) at least 5.73 m b) 7.16 m

17. Answers may vary. Sample answer: a) 3 b)  $y$  is an integer but  $x$  is a power of 10.



18. a) Answers may vary. Sample answer:

x	y
$10^0$	0
$10^1$	1
$10^2$	2
$10^3$	3
$10^4$	4
$10^5$	5
$10^6$	6

b) The graph is linear. The semi-log grid has turned each power into the exponent value.

c) To plot a greater range of values.

19. b) The graph is a line that is increasing, with positive, increasing slope. c) Answers may vary.

20. D

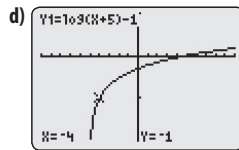
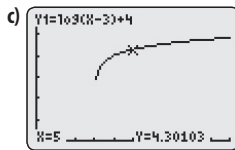
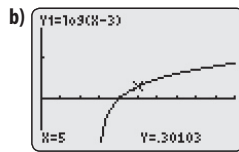
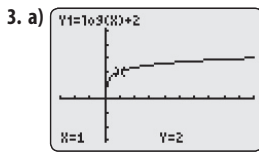
21.  $\sqrt{5}$

### 6.3 Transformations of Logarithmic Functions, pages 338–340

1. a) iv b) ii c) i d) iii

2. a) translate right 2 b) translate left 5 and down 4

c) translate up 1 d) translate left 4 and down 6

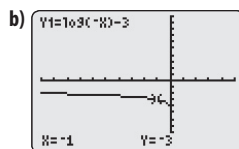
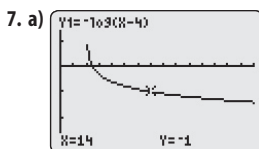
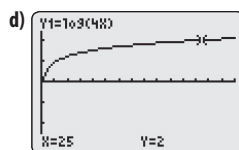
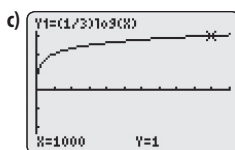
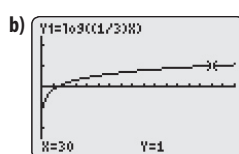
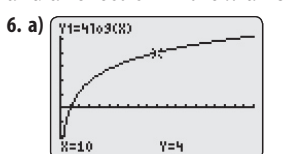


4. a)  $y = 2 \log x$  b)  $y = \log(2x)$  c)  $y = \log\left(\frac{1}{2}x\right)$  d)  $y = \frac{1}{2} \log x$

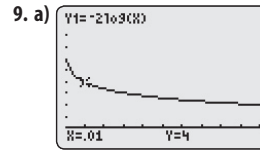
5. a) vertical compression by a factor of  $\frac{1}{2}$

b) horizontal compression by a factor of  $\frac{1}{5}$  and a reflection in the y-axis

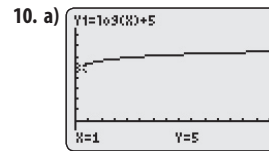
c) horizontal stretch by a factor of 2 and a reflection in the y-axis



8. For  $y = -\log(x - 4)$ : a)  $\{x \in \mathbb{R}, x > 4\}$  b)  $\{y \in \mathbb{R}\}$  c)  $x = 4$   
For  $y = \log(-x) - 3$ : a)  $\{x \in \mathbb{R}, x < 4\}$  b)  $\{y \in \mathbb{R}\}$  c)  $x = 0$



b)  $\{y \in \mathbb{R}, 2 \leq y \leq 6\}$

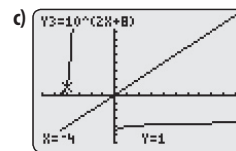
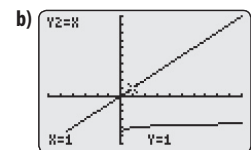
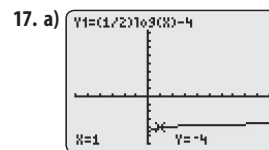
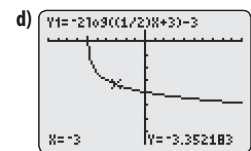
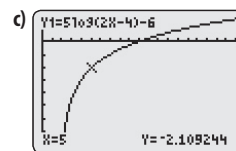
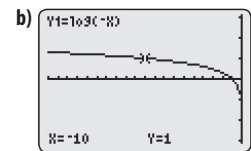
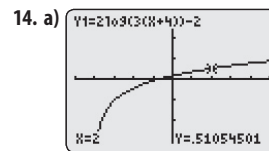


b)  $V_o = 6$ ,  $V_o \approx 6.3$  c)  $V_i = 10^{20}$  d) The domain is the input voltage,  $V_i > 0$ ; the range is the output voltage,  $V_o$ , which can be any real number.

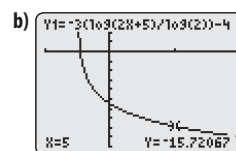
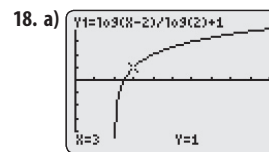
11. Answers may vary. Sample answer: The domain and the range are the same.

12. Answers may vary. Sample answer: No; the domains are different, but the ranges are the same.

13. Answers may vary. Sample answer: The domains are different, but the ranges are the same.

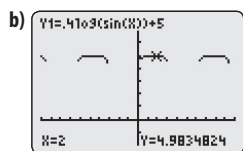


d) i)  $\{x \in \mathbb{R}\}$  ii)  $\{y \in \mathbb{R}, y > 0\}$  iii)  $y = 0$  e)  $y = 10^{2x+8}$





19. a)  $\{t \in \mathbb{R}, n\pi < t < (n+1)\pi, n \in \mathbb{Z}\}, \{V_0 \in \mathbb{R}, V_0 \leq 5\}$



c) Answers may vary.

20.  $x = 64$

### 6.4 Power Law of Logarithms, pages 347–348

1. a) 12 b) 3 c)  $-8$  d)  $-\frac{1}{2}$

2. a)  $\frac{3}{2}$  b)  $\frac{5}{2}$  c) 8 d) 6

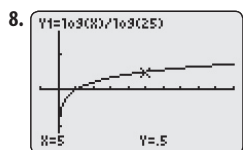
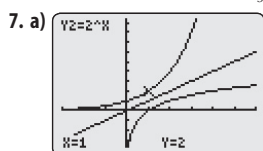
3. a)  $t \doteq 1.66$  b)  $t \doteq 3.43$  c)  $t \doteq 9.01$  d)  $t \doteq 27.62$

4. a) i) \$500 ii) \$572 iii) \$655.40 b) i)  $t \doteq 10.2$  years

ii)  $t \doteq 16.2$  years

5. a) 2.854 b) 1.672 c) 0.356 d)  $-0.558$  e)  $-4.907$  f)  $-7.228$

6. a)  $\log_5 8$  b)  $\log_9 17$  c)  $\log_2 \frac{1}{3}$  d)  $\log_{(x-1)}(x+1)$



9. a)  $x \doteq 4.192$  b)  $x \doteq 3.333$  c)  $x \doteq 1.623$  d)  $x \doteq 8.790$

10. a) \$400; this is the amount when  $t = 0$ .

b) approximately 8 years

11.  $\log(mx) = m \log x$  only when  $m = 1$  and/or when  $x = 1$ .

12.  $\log x^n = (n \log x)$  only when  $n = 1$  and/or when  $x = 1$ .

13. a) 15 b) 15 c) Answers may vary.

14. Answers may vary.

15. a) approximately 6.6 h b)  $\{d \in \mathbb{R}, 0 < d \leq 1000\}$ ,

$\{t \in \mathbb{R}, t \geq 0\}$

18. a)  $\frac{\log_2 9}{\log_2 3}$  b)  $\frac{\log_2 25}{\log_2 10}$

19.  $\log_2(2^{10})^{64} = 640$

20. a)  $A = P(1.00875)^{4t}$  b) i) approximately 19.9 years

ii) approximately 31.5 years

21.  $\frac{2}{\sqrt{3}}$

22. 384

### 6.5 Making Connections: Logarithmic Scales in the Physical Sciences, pages 353–355

1. a) 2 b) approximately 4.5 c) 9 d) approximately 9.8

2. a)  $[H^+] = 10^{-11}$  b)  $[H^+] = 0.001$  c)  $[H^+] \doteq 3.2 \times 10^{-9}$

d)  $[H^+] \doteq 0.0000398$

3. a) Answers may vary. Sample answer: The pH scale varies over several powers of 10. b) Answers may vary.

Sample answer:  $[H^+]$  is very small.

4. a) 5 b) approximately 10.6

5. a) Answers may vary. Sample answer:  $0 < x < 0.003$ ,  $X_{\text{scel}} 1 \times 10^{-4}$ ,  $0 < y < 5$ ,  $Y_{\text{scel}} 1$  b) approximately 2.7

6. a) 100 times b) 100 000 times

7. 10 000 times

8. approximately 15 dB

9. 4

10. a) approximately 199.53 times as intense

b) approximately 15.85 times as intense

11. approximately 794 328 235 times as intense

12. a) approximately 41.69 times brighter

b) approximately  $-11.61$

14. Answers may vary. Sample answer: The absolute magnitude takes distance into consideration.

15. a) i) approximately 158.5 times ii) 100 000 times

b) closest star is Biffidus-V, next is Cheryl-XI, farthest away is Roccolus-III

16.–18. Answers may vary.

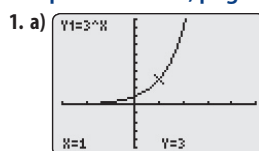
19. B

20.  $\sqrt{2}s$

21. B

22. D

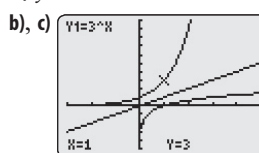
### Chapter 6 Review, pages 356–357



i)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y > 0\}$  ii) no  $x$ -intercept iii) 1

iv) positive for all intervals v) increasing for all intervals

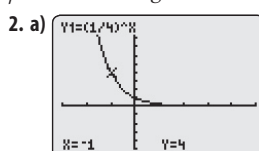
vi)  $y = 0$



c)  $\{x \in \mathbb{R}, x > 0\}; \{y \in \mathbb{R}\}; x$ -intercept 1; no  $y$ -intercept;

$f^{-1}$  is positive for  $x > 1$  and negative for  $0 < x < 1$ ;

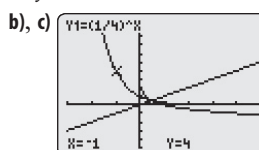
$f^{-1}$  is increasing for all intervals; vertical asymptote:  $x = 0$



i)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y > 0\}$  ii) no  $x$ -intercept iii) 1

iv) positive for all intervals v) decreasing for all intervals

vi)  $y = 0$



c)  $\{x \in \mathbb{R}, x > 0\}; \{y \in \mathbb{R}\}; x$ -intercept 1; no  $y$ -intercept;

$f^{-1}$  is positive for  $0 < x < 1$  and negative for  $x > 1$ ;

$f^{-1}$  is decreasing for all intervals; vertical asymptote  $x = 0$

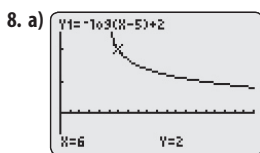
3. a)  $\log_4 64 = 3$  b)  $\log_3 28 = x$  c)  $\log_6 y = 3$  d)  $\log_2 512 = 9$

4. a)  $2^7 = 128$  b)  $b^x = n$  c)  $3^5 = 243$  d)  $b^{19} = 4$

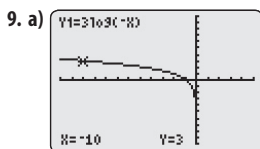
5. approximately 5.6

6. a) 4 b) 4 c)  $-2$  d)  $-6$

7.  $\log_x x = 1, x > 0, x \neq 1$



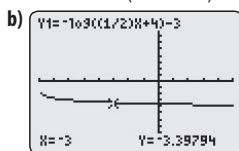
b) i)  $\{x \in \mathbb{R}, x > 5\}$  ii)  $\{y \in \mathbb{R}\}$  iii)  $x = 5$



b) i)  $\{x \in \mathbb{R}, x < 0\}$  ii)  $\{y \in \mathbb{R}\}$  iii)  $x = 0$

11. Answers may vary. Sample answer:

a)  $y = -\log\left(\frac{1}{2}x + 4\right) - 3$

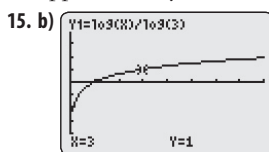


c) i)  $\{x \in \mathbb{R}, x > -8\}$  ii)  $\{y \in \mathbb{R}\}$  iii)  $x = -8$

12. a) 15 b) -6 c) 3 d) 8

13. a)  $x \approx 2.579$  b)  $x \approx -1.515$  c)  $x \approx 1.661$  d)  $x \approx 0.322$

14. approximately 1.3 mm



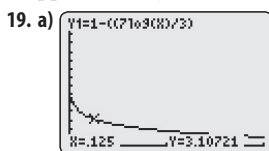
16. There is less hydronium ion concentration in Chemical B.

17. No; a great earthquake is 10 000 times as intense as a light earthquake.

18. a)  $3 (n \approx 3.1)$

b) approximately  $\frac{1}{372\,717}$  ( $T \approx 2.683 \times 10^{-6}$ )

c) approximately 138 934 times as much



b)  $\{T \in \mathbb{R}, 0 < T \leq 1\}, \{n \in \mathbb{R}, n \geq 1\}$

20. Answers may vary.

### Chapter 6 Practice Test, pages 358–359

1. D

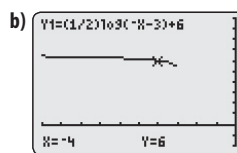
2. A

3. B

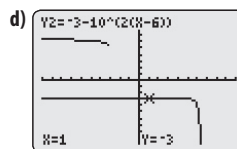
4. C

5. a) 1.839 b) 2.163

6. a) Compress vertically by a factor of  $\frac{1}{2}$ , translate 3 left, translate 6 up, and reflect in the y-axis.



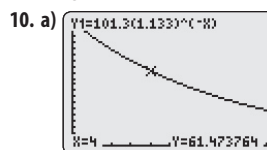
c) i)  $\{x \in \mathbb{R}, x < -3\}$  ii)  $\{y \in \mathbb{R}\}$  iii)  $x = -3$



8. approximately 11 years

9. a)  $\text{pH} \approx 11.5$  b) approximately 0.000 32  $< V < 0.01$

c) vinegar acidic, ammonia alkaline



b) approximately 54.3 kPa c) approximately 1.4 km

11. a) approximately 110 dB b) approximately 1995 times

## CHAPTER 7

### Prerequisite Skills, pages 362–363

1. product law a)  $x^9$  b)  $6a^5b^4$  c)  $x^4y^5$  d)  $6r^2s^2$

e)  $5q^2r$  f)  $v^3w^4$  g)  $4ab^4$

2. quotient law a)  $k^4$  b)  $-6n$  c)  $6x^3y$  d)  $2ab^2$  e)  $\frac{q^2}{2r}$  f)  $v^2w^2$  g)  $b^4$

3. power law a)  $w^8$  b)  $4u^2v^6$  c)  $a^6b^3$  d)  $x^6y^6$  e)  $8uw^6$  f)  $a^2b^8$  g)  $rs^6$

4. a)  $\frac{2a^2}{b}$ , product and quotient laws b)  $-108k^9m^{13}$ , product and power laws c)  $32y^5$ , product, quotient, and power laws

d)  $a^4b$ , product, quotient, and power laws

5. a)  $x = -6$  or  $x = 4$  b)  $x = 2.5$  or  $x = 2$  c)  $x = \frac{1}{2}$  or  $x = -\frac{7}{2}$

d)  $q = 5$  or  $q = -4$  e)  $b = \frac{1}{3}$  or  $b = -\frac{1}{3}$  f)  $y = -\frac{1}{2}$  or  $y = -\frac{1}{4}$

g)  $x = 2$  or  $x = -1$  h)  $r = -3$  or  $r = \frac{1}{2}$  i)  $q = -4$  or  $q = -1$

6. a)  $y = -3 \pm \sqrt{14}$  b)  $q = \frac{1 \pm \sqrt{41}}{4}$  c)  $x = \frac{-1 \pm \sqrt{5}}{2}$

d)  $a = \frac{-1 \pm \sqrt{21}}{2}$  e)  $x = \frac{1 \pm \sqrt{29}}{2}$  f)  $r = \frac{5 \pm \sqrt{13}}{2}$

g)  $m = \frac{-3 \pm \sqrt{3}}{2}$  h)  $a = \frac{3 \pm \sqrt{57}}{6}$

7. a)  $2\sqrt{2}$  b)  $4\sqrt{5}$  c)  $3\sqrt{2}$  d)  $9\sqrt{2}$  e)  $5\sqrt{3}$

f)  $2(\sqrt{3} + 1)$  g)  $1 \pm \sqrt{6}$

8. a) 6 b)  $\frac{5}{4}$  c) 6 d) 4 e) 10 f) 4 g) 4 h) -2

9. a) 2.096 b) 2.322 c) 3.907 d) 2.044 e) 1.585 f) 1.893

g) 2.161 h) 4.248

10. a) 0.86 b) 1.29 c) 0.36 d) -0.17 e) 0.63 f) 0.53 g) 3.17 h) 1.30

### 7.1 Equivalent Forms of Exponential Equations, pages 368–369

1. a)  $2^{12}$  b)  $2^9$  c)  $2^{-6}$  d)  $2^{\frac{\log 14}{\log 2}}$

2. a)  $3^6$  b)  $3^{-4}$  c)  $3^{\frac{1}{\log 3}}$  d)  $3^{\frac{\log \frac{1}{2}}{\log 3}}$

3. Answers may vary. Sample answers: a)  $4^4$  b)  $16^2$

4. a)  $4^3$  b)  $4^{\frac{33}{8}}$  c)  $4^{\frac{16}{3}}$  d)  $2^{\frac{16}{3}}$

5. a)  $x = 3$  b)  $x = -2$  c)  $w = 3$  d)  $m = \frac{7}{4}$

6. a)  $x = -3$  b)  $x = 6$  c)  $y = \frac{11}{4}$  d)  $k = 9$

7. a)  $x = 5$  b)  $x = 5$  c) Answers may vary.

8. a)–d) Answers may vary.

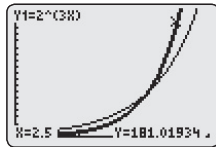
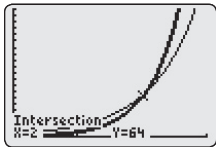
9. Answers may vary.

10. a)  $x = -4$  b)  $k = -2$

11. a)  $x = \frac{1}{\log 2}$

12.  $10 = b^{\frac{1}{\log b}}$ ,  $b > 0$

15. a) i)  $x > 2$  ii)  $x > -\frac{11}{2}$  b) Answers may vary. Sample answer for inequality i): Graph each side of the inequality as a separate function. Find their point of intersection. Test a point to the right of the point of intersection to ensure that the inequality is true.



c) Answers may vary.

16. a) i)  $2^x > x^3$  for  $0 < x < 1.37$ ;  $x^3 > 2^x$  for  $x > 1.37$  (correct to 2 decimal places). ii)  $1.1^x > x^{10}$  for  $0 < x < 1.01$ . b) for  $x > 1.52$

## 7.2 Techniques for Solving Exponential Equations, pages 375–377

1. a) iii) b) i) c) ii) d) iv)

2. a) 10.24 b) 11.53 c) 58.71 d) 18.91 e) -2.80 f) -0.55 g) -15.63 h) -3

3. a) approximately 35.75 m b) approximately 10.3 min c) No.

4. a)  $x = \frac{\log 3}{\log 3 - \log 2}$  b)  $x = \frac{2 \log 5}{\log 5 - \log 4}$

c)  $x = \frac{\log 8 + \log 3}{\log 3 - \log 8}$  d)  $x = \frac{\log 7 + 2 \log 4}{\log 4 - 2 \log 7}$

5. a) 2.710 b) 14.425 c) -3.240 d) -1.883

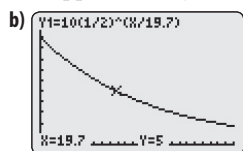
6. a)  $a = 1$ ,  $b = 1$ ,  $c = -6$  b)  $x = 1$  c)  $2^x = -3$

7. a)  $a = 1$ ,  $b = -2$ ,  $c = -5$  b)  $x = \frac{\log(1 + \sqrt{6})}{\log 8}$

c)  $8^x = 1 - \sqrt{6}$

8. a) approximately 21.3 min b) approximately 92.06 min

9. a) approximately 19.7 min



c) i) The graph is decreasing faster (shorter time). c) ii) The graph is decreasing at a slower rate (longer time to decay). d) i) steeper slope (negative), more would decay in the same amount of time ii) slope is not as steep, less to decay in same amount of time

10. No.

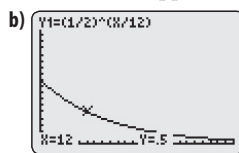
11. a)  $x = 1$  or  $x \approx 1.26$  b)  $x \approx 0.16$  c)  $x = -1$  d)  $x = 1$  e) no solution f)  $x \approx 1.29$

13. a) approximately 66.4 h b) Answers may vary.

14. approximately 6.44 days

15. a) 6 years b) approximately 10 years

16. a) i) 12 s ii) approximately 40 s



c) Answers may vary.

18. a)  $y = \left(\frac{1}{2}\right)^{\frac{t}{20}}$  b)  $\{t \in \mathbb{R}, t \geq 0\}$ ,  $\{y \in \mathbb{R}, 0 < y \leq 1\}$

19. Answers may vary.

20. D

21. C

22. 11

## 7.3 Product and Quotient Laws of Logarithms, pages 384–386

1. a)  $\log 54$  b)  $\log 8$  c)  $\log_3 21$  d)  $\log_5 2$

2. a) 1.732 b) 0.903 c) 2.771 d) 0.431

3. a)  $\log(2xyz)$ ,  $x > 0$ ,  $y > 0$ ,  $z > 0$

b)  $\log_2\left(\frac{3ab}{2c}\right)$ ,  $a > 0$ ,  $b > 0$ ,  $c > 0$

c)  $\log\left(\frac{m^2 n^3}{y^4}\right)$ ,  $m > 0$ ,  $n > 0$ ,  $y > 0$

d)  $\log(u^2 v \sqrt{w})$ ,  $u > 0$ ,  $v > 0$ ,  $w > 0$

4. a) 2 b) 2 c) 2 d) 3

5. a) 3 b) 4 c) 3 d) -2

6. a) 2 b) approximately 2.301

7. a)  $\log_7 c + \log_7 d$  b)  $\log_3 m - \log_3 n$  c)  $\log u + 3 \log v$

d)  $\log a + \frac{1}{2} \log b - 2 \log c$  e)  $\log_2 2 + \log_2 5 + 1 + \log_2 5$

f)  $\log_5(25 \times 2) = 2 + \log_5 2$

8. Answers may vary.

9. a)  $\frac{3}{2} \log x$ ,  $x > 0$  b)  $\log m$ ,  $m > 0$  c)  $\frac{8}{3} \log k$ ,  $k > 0$

d)  $\frac{9}{2} \log w$ ,  $w > 0$

10. a)  $\log(x + 2)$ ,  $x > 2$  b)  $\log(x + 4)$ ,  $x > -3$

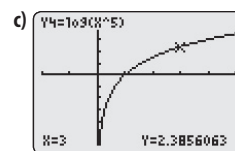
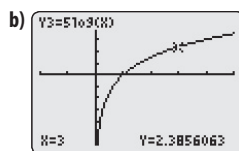
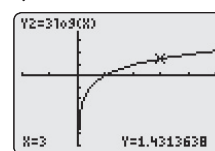
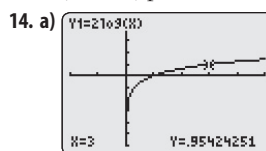
c)  $\log\left(\frac{x + 2}{2}\right)$ ,  $x > 3$  d)  $\log\left(\frac{x + 4}{x - 3}\right)$ ,  $x > 3$

11. a)  $V_o = \log\left(\frac{V_2}{V_1}\right)$  b) i)  $V_o = 1$  ii)  $V_o = 2$  iii)  $V_o = 0$

12. a) Translate  $y = \log x$  up  $(1 + \log n)$  units.

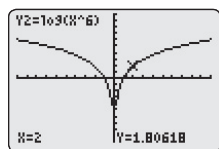
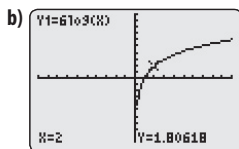
b) Answers may vary.

13. no,  $n > 0$ , positive integers only



14. d)  $p(x) = q(x)$ ; power law

15. a)  $q(x) = \log x^6$



c) Answers may vary. Sample answer: The graph has values for  $x < 0$  and  $x > 0$  because  $x^6$  is an even power; the graph has a discontinuity at  $x = 0$ .

16. Answers may vary.

18. a) approximately 6.7 years b) i) B ii) B c) Answers may vary.

22.  $\sqrt{2}$

23.  $-1$

24. B

25.  $\frac{c}{k} - \frac{c}{k+n}$

### 7.4 Techniques for Solving Logarithmic Equations, pages 391–392

1. a)  $x = 12$  b)  $x = 75$  c)  $p = 38$  d)  $w = 17$

e)  $k = 108$  f)  $n = 50$

2. a)  $x = 5$  b)  $x = 21$  c)  $k = 27$  d)  $x = 25$  e)  $t = \frac{9}{8}$  f)  $n = 2$

3. a)  $x = 2 + \sqrt{14}$  b)  $x = 4$  c)  $v = \frac{533}{33}$  d)  $y = 1$

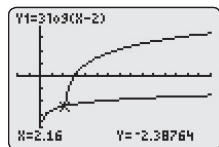
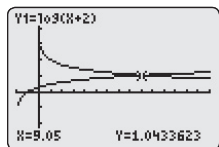
e)  $k = 3$  f)  $p = 1$

5. a)  $x = -2$  or  $x = 5$  b)  $x = -50$  or  $x = 2$

6. a)  $x = \frac{5}{511}$  b)  $k = \frac{1}{2}$

7. a)  $x \doteq 9.05$

b)  $x \doteq 2.16$

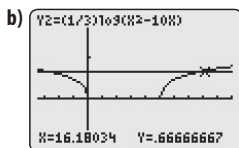


8. a) approximately 119.54 dB b)  $1 \times 10^{-10}$  W/m<sup>2</sup> c) 1 W/m<sup>2</sup>

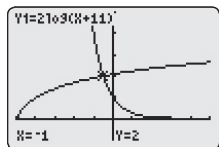
9. a) No. b) Yes.

10. a) 2 b)  $\frac{1}{16}$

11. a)  $w = 5 \pm 5\sqrt{5}$



12.  $x = -1$ ; graph each side of the equation as a function and find the point of intersection.



14. 44

15.  $\pm 3\sqrt{5}$  or approximately 6.7 or  $-6.7$

16. D

### 7.5 Making Connections: Mathematical Modelling With Exponential and Logarithmic Equations, pages 404–407

1. in June of 2037

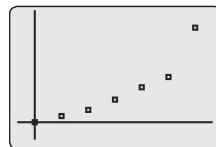
2. in approximately the year 1774

3. a) for  $P = 1006(1.016)^t$  i)  $P \doteq 4920$  ii)  $t \doteq 188.4$  years; for  $P = 1000 \times 2^{\frac{t}{43.5}}$  i)  $P \doteq 4921$  ii)  $t \doteq 188$  years b) quite close

4. Answers may vary.

5. Answers may vary. Sample answer: Rural Ontario Investment Group takes a little less time to make \$80 000 (7.35 years), so it could be considered. Muskoka Guaranteed Certificate takes the same amount of time, so it does not need to be considered.

6. a) Answers may vary. Sample answer: increasing in a curved pattern



b) i)  $y = 4.21x - 3.07$  ii)  $y \doteq 0.86x^2 - 0.93x + 1.2$

iii)  $y = 1.37(1.65)^x$

c) Answers may vary. Sample answer: exponential

d) Answers may vary. Sample answers:

i) approximately 207.8 m<sup>2</sup>

ii)  $t \doteq 12.44$  min e) Answers may vary. Sample answer: circle

7. a)  $A = 1000(1.02)^{4t}$  b) approximately \$1372.79

c) approximately 8.75 years

8. a)  $A = 1000(1.02)^{4t} - 50$  if  $t < 4$ .

b) shifts the graph down by 50

9. Answers may vary.

10. a)  $1000 = \frac{I_2}{I_1}$  b) by 10 000 000 (or  $10^7$ )

12–14. Answers may vary.

15. 48 km/h

16.  $A = \frac{\pi}{2} - 1$

17.  $z$  is increased by 12.5%.

### Chapter 7 Review, pages 408–409

1. a)  $4^3$  b)  $4^1$  c)  $4^{-2}$  d)  $4^{\frac{5}{2}}$

2. a)  $5^{\frac{\log 20}{\log 5}}$  b)  $5^{\frac{\log 0.8}{\log 5}}$

3. a)  $x = -\frac{3}{2}$  b)  $x = -18$

4. a) approximately 5 min b) approximately 21.6 min

5. a)  $x = \frac{2 \log 3}{\log 3 - \log 5}$  b)  $k = \frac{2 \log 2 + \log 3}{\log 2 - \log 3}$

6. a)  $x \doteq -4.301$ ,  $k \doteq -6.129$

7. a)  $x = \frac{\log 5}{\log 4}$  b)  $x = 2$  or  $x = \frac{\log 3}{\log 2}$

8. a) approximately 2.5 years b) approximately 8.3 years

9. a) 3 b) 2 c) 2 d) approximately 2.05

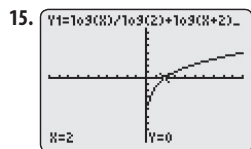
10. a)  $\log_7 2$  b)  $\log\left(\frac{3a^2b}{\sqrt{c}}\right)$ ,  $a > 0$ ,  $b > 0$ ,  $c > 0$

11. a)  $2 \log a + \log b + \log c$  b)  $\log k - \frac{1}{2} \log m$

12. a)  $\log\left(\frac{2}{m-3}\right)$ ,  $m > 3$  b)  $\log\left(\frac{x+5}{x-4}\right)$ ,  $x > 4$ ,  $x > -5$

13. a)  $x = 45$  b)  $x = 5$

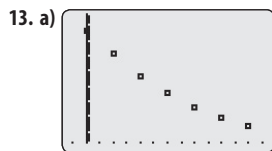
14.  $x = 2$



16. **a)** approximately 11.6 h **b)** approximately 66%  
 17. **a)**  $A = 500(1.033)^{2t}$  **b)** approximately \$691.79  
**c)** approximately 10.67 years  
**d) i)** same shape translated up by 5 **ii)** above the original function and increasing at a faster rate

**Chapter 7 Practice Test, pages 410–411**

1. C  
 2. A  
 3. D  
 4. C  
 5. 2  
 6. **a)**  $x = 8$  **b)**  $x = \frac{16}{3}$  **c)**  $x = 15$  **d)**  $x = 2.8$   
 8. **a)**  $\log(x + 1) = 5 \log x - 2$  **b)**  $x \approx 3.37$   
 9. **a)** approximately 35 min **b)** approximately 232.5 min (or 3 h 52 min 30 s)  
 10. approximately 9.9 min  
 11. **a)**  $x = 3$   
 12. **a)** Yes. **b)** Yes. **c)** Answers may vary.



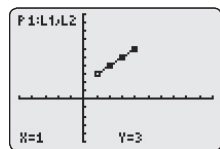
- b)**  $y \approx 0.17x^2 - 5.34x + 97.93$  **c)**  $y \approx 95.67(0.96)^x$   
**d)** Answers may vary. Sample answer: Exponential, because if predicting values for  $x > 15.74$ , the quadratic gives increasing values, which is not realistic.  
**e) i)** approximately 49.6°C **ii)** approximately 21.1 min  
**f)** Answers may vary. Sample answer: The temperature of the room was constant.

**CHAPTER 8**

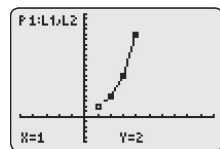
**Prerequisite Skills, pages 414–415**

1. **b)** Pattern A: linear, Pattern B: exponential, Pattern C: quadratic  
 2. **a)**

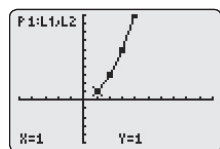
Pattern A



Pattern B



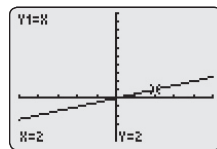
Pattern C



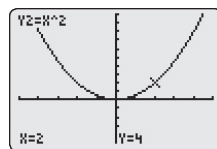
- b)** Yes.

3. **A:**  $y = x + 2$ ; **B:**  $y = 2^x$ ; **C:**  $y = \frac{1}{2}x^2 + \frac{1}{2}x$

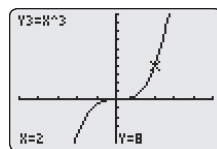
4. **a)**  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$



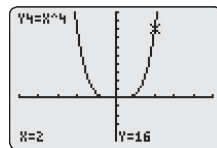
**b)**  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 0\}$



**c)**  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

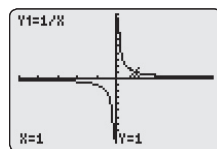


**d)**  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 0\}$

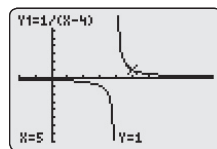


5. **a)** odd **b)** even **c)** odd **d)** even

7. **a)**  $\{x \in \mathbb{R}, x \neq 0\}, \{y \in \mathbb{R}, y \neq 0\}$



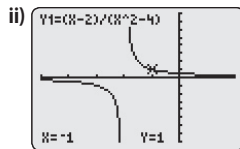
**b)**  $\{x \in \mathbb{R}, x \neq 4\}, \{y \in \mathbb{R}, y \neq 0\}$



8. **a)**  $u(x) = \frac{1}{x + 2}, x \neq 2, x \neq -2$

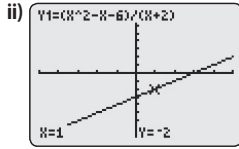
**b)**  $v(x) = x - 3, x \neq 2$

9. **a) i)**  $\{x \in \mathbb{R}, x \neq 2, x \neq -2\}, \{y \in \mathbb{R}, x \neq 0, y \neq \frac{1}{4}\}$



**iii)** asymptotes:  $x = -2, y = 0$ ; hole:  $(2, \frac{1}{4})$

b) i)  $\{x \in \mathbb{R}, x \neq -2\}, \{y \in \mathbb{R}, y \neq -5\}$



iii) hole:  $(-2, -5)$

11. a) iv b) iii c) ii d) i

12. Answers may vary. Sample answers:

i)  $0 \leq t \leq 8 \text{ s}, -5 \text{ cm} \leq d \leq 5 \text{ cm}$

ii)  $0 \leq t \leq 3 \text{ s}, 0 \text{ m} \leq d \leq 5 \text{ m}$

iii)  $0 \leq t \leq 10 \text{ s}, -2 \text{ m} \leq d \leq 2 \text{ m}$

iv)  $0 \leq t \leq 10 \text{ s}, 0 \text{ m} \leq d \leq 3 \text{ m}$

13. a)  $y = x + 2$  b)  $y = \frac{x-3}{4}$  c)  $y = \pm\sqrt{x+5}, x \geq -5$

d)  $y = \frac{1}{x} - 1, x \neq 0$

14. The inverses of parts a), b), and d) are functions, since they pass the vertical line test.

### 8.1 Sums and Differences of Functions, pages 416–428

1. a) i) blue ii) red iii) yellow

b) i)  $y = 3x$  ii)  $y = x^2 - 1$  iii)  $y = 2^x$

2. a)  $y = 3x + x^2 - 1 + 2^x$  b) i) 71 ii) 117

3. a) i)  $y = 6x + 7$  ii)  $y = 4x - 7$  iii)  $y = -4x + 7$

b) i)  $y = -3x + 14$  ii)  $y = -x - 4$  iii)  $y = x + 4$

c) i)  $y = x^2 + 5$  ii)  $y = x^2 + 3$  iii)  $y = -x^2 - 3$

d) i)  $y = -3x^2 + 7x - 7$  ii)  $y = -3x^2 + x + 7$

iii)  $y = 3x^2 - x - 7$

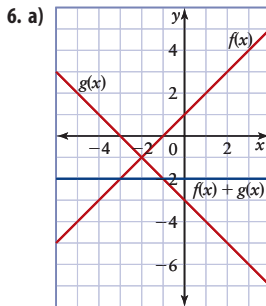
4. a)  $h(x) = 7x + 1, h(2) = 15$  b)  $j(x) = x + 5, j(-1) = 4$

c)  $k(x) = -x - 5, k(0) = -5$

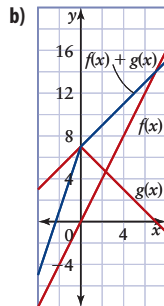
5. a)  $h(x) = -4x^2 + 2x + 2, h(-3) = -40$

b)  $j(x) = -4x^2 - 2x + 8, j(0) = 8$

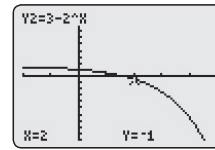
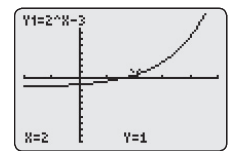
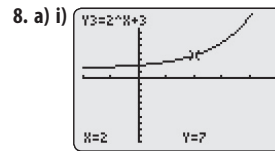
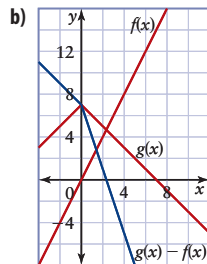
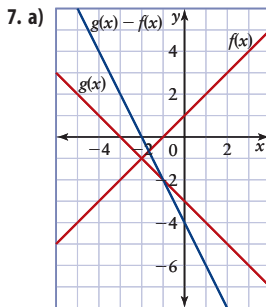
c)  $k(x) = 4x^2 + 2x - 8, k(3) = 34$



$\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$



$\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$



b) i) 3 units up ii) 3 units down

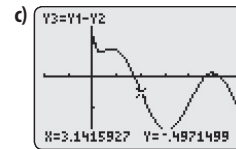
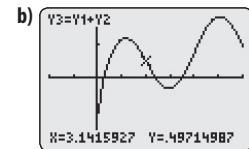
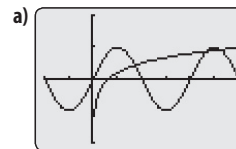
iii) reflection in the  $x$ -axis and 3 units up

c) i)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y > 3\}$

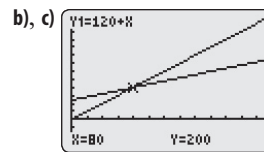
ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y > -3\}$

iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y < 3\}$

9. Window variables:  $x \in [-\pi, 3\pi], Xscl \frac{\pi}{2}, y \in [-2, 2]$

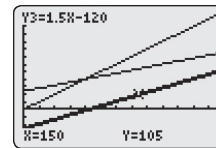


10. a) i)  $C = 120 + b$  ii)  $R = 2.5b$



The break-even point is the point at which the revenue and cost are equal. When the vendor has sold 80 hotdogs, the cost and the revenue are both equal to \$200.

d)  $P(b) = 1.5b - 120$



e)  $C(b): \{b \in \mathbb{Z}, 0 \leq b \leq 250\}, \{C \in \mathbb{R}, 120 \leq C \leq 370\}$

$R(b): \{b \in \mathbb{Z}, 0 \leq b \leq 250\}, \{R \in \mathbb{R}, 0 \leq R \leq 625\}$

$P(b): \{b \in \mathbb{Z}, 0 \leq b \leq 250\}, \{P \in \mathbb{R}, -120 \leq P \leq 255\}$

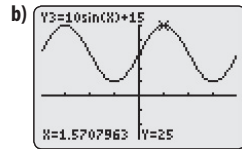
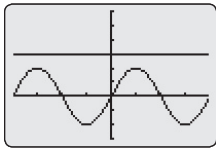
f) \$255

11. a) i) If  $C = 100 + b$ , then the vendor only needs to sell about 67 hotdogs to break even.

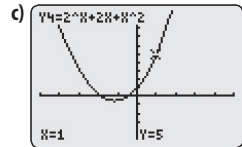
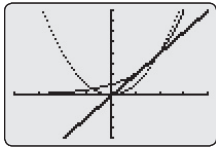
ii) If  $C = 120 + 0.9b$ , then the potential daily profit becomes \$280.

b) Answers may vary. Sample answer: Choose to reduce the variable cost.

12. a) Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ ,  $X_{\text{scl}} \frac{\pi}{2}$ ,  $y \in [-15, 30]$ ,  $Y_{\text{scl}} 5$



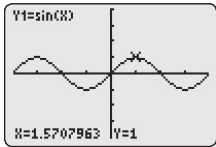
c)  $\{t \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, 5 \leq y \leq 25\}$  d) i) 5 ii) 25 iii) 15  
 13. a) same b) Window variables:  $x \in [-4, 4]$ ,  $y \in [-4, 8]$



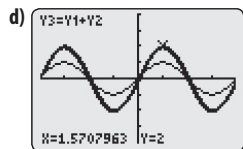
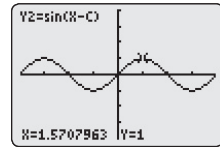
d)  $f(x) = 2^x$  e) Answers may vary. The rate of change of the exponential function is continuously increasing at a greater rate than the other component functions.

14. a) Yes. b) No. c) The commutative property holds true for the sum of two functions, but not the difference of two functions.

15. a)



c) same



e)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, -2 \leq y \leq 2\}$

f) i) shifted to the right and amplitude multiplied by  $\sqrt{2}$

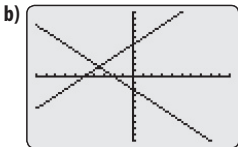
ii) horizontal line iii) same as when  $c = 0$

g) i)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, -\sqrt{2} \leq y \leq \sqrt{2}\}$

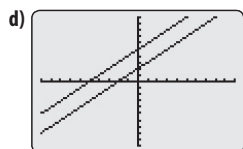
ii)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y = 0\}$  iii)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, -2 \leq y \leq 2\}$

h) Answers may vary.

16. a)  $y = -x - 2$



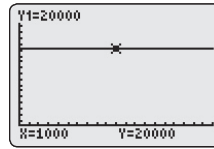
c)  $y = 3$



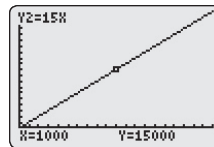
e)  $y = 3$ , same

f) Subtracting the functions is the same as adding the opposite.

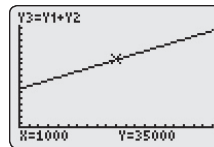
18. a) \$20 000; not affected by the number of games;  
 $Y_1 = 20\,000$



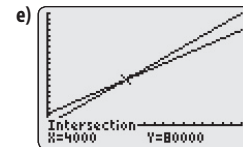
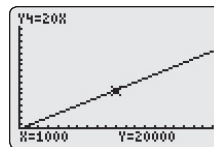
b) \$15/game; cost increases per game at a constant rate;  
 $Y_2 = 15x$



c) total operating costs;  $Y_3 = 20\,000 + 15x$

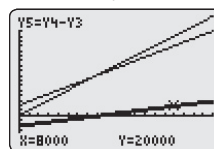


d) revenue increasing at a constant rate;  $Y_4 = 20x$



Break-even point: (4000, 80 000), cost equals revenue

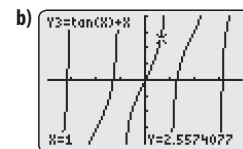
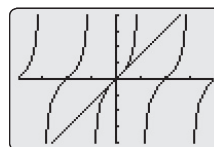
f) profit;  $Y_5 = 5x - 20\,000$



g) i) loss ii) break-even iii) profit

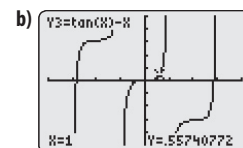
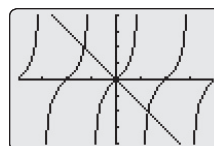
h) i) move to the left ii) move to the right

20. a) Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ ,  $X_{\text{scl}} \frac{\pi}{2}$ ,  $y \in [-4, 4]$



d) infinite number e)  $g(x)$  intersects  $h(x)$  at the point of inflection due to the variable vertical translation.

21. a) Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ ,  $X_{\text{scl}} \frac{\pi}{2}$ ,  $y \in [-4, 4]$



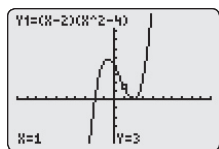
d) infinite number

e)  $g(x)$  intersects  $h(x)$  at the point of inflection due to the variable vertical translation.

22. Yes  
 23. The sum of two even functions is even.  
 24. B  
 25. D  
 26. 4  
 27. negative reciprocals:  $A = -C$ ; reciprocals:  $A = C$

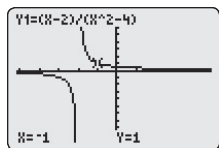
### 8.2 Products and Quotients of Functions, pages 429–438

1. a) even: A and B; odd: A and C or B and C  
 b) Two combinations multiply to form an odd function.  
 2. odd  
 4. a)  $y = x^3 - 2x^2 - 4x + 8$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$

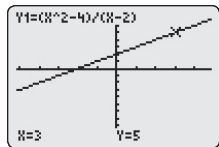


b)  $y = \frac{1}{x+2}$ ,  $x \neq -2$ ,  $x \neq -2$ ,  $\{x \in \mathbb{R}, x \neq -2\}$ ,  
 $\{y \in \mathbb{R}, y \neq \frac{1}{4}, y \neq 0\}$

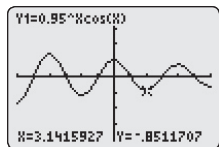
hole:  $(2, \frac{1}{4})$ , asymptotes:  $x = -2$  and  $y = 0$



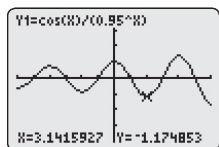
c)  $y = x + 2$ ,  $x \neq 2$ ,  $\{x \in \mathbb{R}, x \neq 2\}$ ,  $\{y \in \mathbb{R}, y \neq 4\}$ , hole:  $(2, 4)$



5. a)  $y = 0.95^x \cos x$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$



b)  $y = \frac{\cos x}{0.95^x}$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$



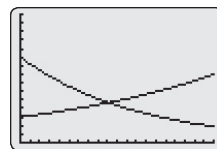
c)  $y = \frac{0.95^x}{\cos x}$ ,  $\cos x \neq 0$ ,  $\left\{x \in \mathbb{R}, x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}\right\}$

$\{y \in \mathbb{R}, y \neq 0\}$ , asymptotes:  $x = \frac{(2n+1)\pi}{2}$ ,  $n \in \mathbb{Z}$ , and  $y = 0$



6. a) Both functions are exponential, with fish increasing and food decreasing.

Window variables:  $x \in [0, 20]$ ,  $y \in [0, 1500]$ , Yscl 100

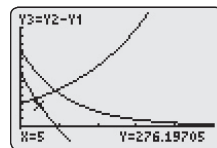


b)  $P(t): \{t \in \mathbb{R}\}$ ,  $\{P \in \mathbb{R}, P \geq 0\}$

$F(t): \{t \in \mathbb{R}\}$ ,  $\{F \in \mathbb{R}, F \geq 0\}$

c) (9.11, 467.88); In 9.11 years, the number of fish and the amount of fish food both equal 467.88.

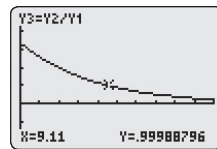
d) Answers may vary. Sample answer: The amount of fish food minus the number of fish. When the function is positive, there is a surplus of food. When the function is negative, there is not enough food.



e) 9.11, same

f) Answers may vary. Sample answer:  $P(t)$  should start to decrease since the amount of food is decreasing.

7. a) Answers may vary. Sample answer: Ratio of food to fish. If the function is greater than one, there is more food. The graph is decreasing.

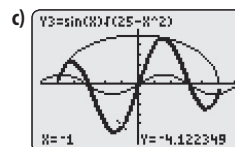
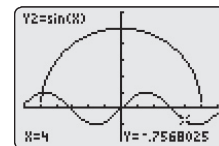
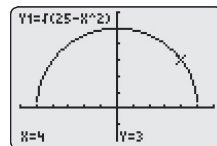


b) approximately (9.11, 1); After 9.11 years, the amount of food is equal to the number of fish.

c) plenty of food, enough food, not enough food

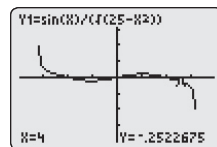
8. a) semi-circle, even

b) odd



d)  $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$ ,  $\{y \in \mathbb{R}, -4.76 \leq y \leq 4.76\}$

9. a) odd;  $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$ ,  $\{y \in \mathbb{R}\}$



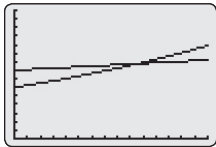


b) odd;  $\{x \in \mathbb{R}, -5 \leq x \leq 5, x \neq -\pi, 0, \pi\}, \{y \in \mathbb{R}\}$

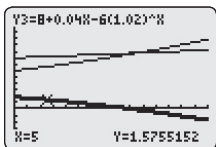


10. a)  $P(t)$  is exponential and increasing, while  $F(t)$  is linear and increasing.

Window variables:  $x \in [0, 30], Xscl 2, y \in [0, 15]$

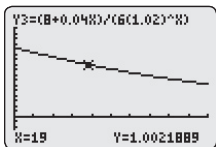


b)  $y = 8 + 0.04t - 6(1.02)^t$ ; Answers may vary. Sample answers: In 2008, there is a surplus, since the function is positive. After 2019, there will be a food shortage.



c) (0, 2); In 2000, the maximum is 2, which is the amount of the surplus of food.

11. a) decreasing



b) 2000; yes

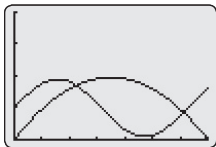
c) When  $\frac{F(t)}{P(t)} > 1$ , there is a surplus of food. When  $\frac{F(t)}{P(t)} < 1$ , there is a shortage of food. For Terra, there is a surplus of food before 2019 and a shortage of food after 2019.

12. Answers may vary.

13. a)  $p_{\text{Ask}}(t)$  is periodic, with domain  $\{t \in \mathbb{R}, t \geq 0\}$  and range  $\{p_{\text{Ask}} \in \mathbb{R}, 0.05 \leq p_{\text{Ask}} \leq 0.95\}$ .

$p_{\text{Yes}}(t)$  is quadratic, increasing and then decreasing, with domain  $\{t \in \mathbb{R}, 0 \leq t \leq 7\}$  and range  $\{p_{\text{Yes}} \in \mathbb{R}, 0 \leq p_{\text{Yes}} \leq 0.98\}$ .

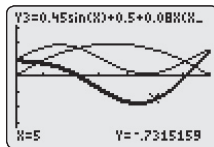
Window variables:  $x \in [0, 7], y \in [0, 2], Yscl 0.5$



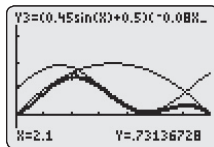
b) at the maximum values of  $p_{\text{Ask}}(t)$ , which occur at  $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$  days after the dance (approximately 1.6, 7.9, 14.1,  $\dots$  days after the dance)

c) at the maximum value of  $p_{\text{Yes}}(t)$ , which occurs 3.5 days after the dance

d) Answers may vary. Sample answer: The zeros of this graph are the points of intersection of the graphs of  $p_{\text{Yes}}$  and  $p_{\text{Ask}}$ .



14. a)  $y = -0.036t^2 \sin t + 0.252t \sin t - 0.04t^2 + 0.28t$



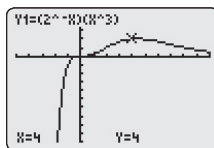
b) There is approximately a 73% chance of Carlos and Keiko agreeing to date 2.1 days after the dance.

c) after 7 days d) Answers may vary.

15. Yes.

16. No.

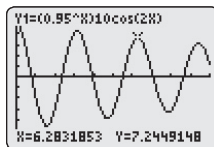
17.  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 4.04\}, x$ -intercept 0,  $y$ -intercept 0;  $x < 0$ , function is negative and increasing;  $0 < x < 4.33$ , function is positive and decreasing;  $x > 4.33$ , function is positive and decreasing; as  $x \rightarrow \infty, y \rightarrow 0$ , and as  $x \rightarrow -\infty, y \rightarrow -\infty$



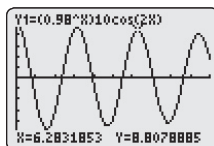
19. a) i) even ii) even iii) even b) i) even ii) odd iii)  $[f(x)]^n$  is even if  $n$  is even and odd if  $n$  is odd when  $f(x)$  is odd.

20. a) 2.45 cm

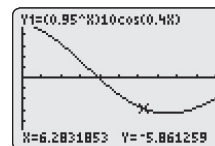
b) original function



i) air resistance reduced



ii) pendulum lengthened



21. 10

22.  $\frac{60}{7}$

23.  $\frac{5}{2}$

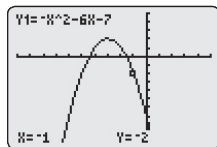
24. 35 cm

### 8.3 Composite Functions, pages 429–449

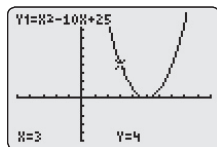
1. a)  $y = -x^2 - 6x - 7$  b)  $y = x^2 - 10x + 25$  c)  $y = x$

d)  $y = x^4 + 12x^3 + 60x^2 + 144x + 144$  e)  $y = x$

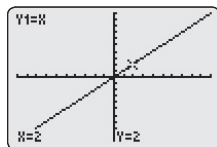
2. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 2\}$



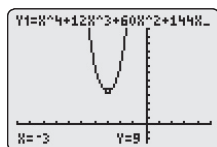
b)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 0\}$



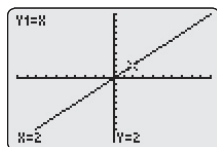
c)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$



d)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 9\}$

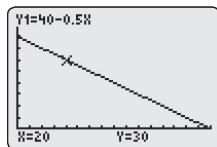


e)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$



4. a)  $-\frac{1}{3}$  b)  $-5$

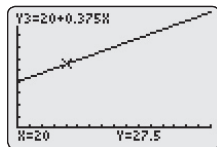
5. a) decreasing at a constant rate



b) 40% c) decreasing by 0.5%/day

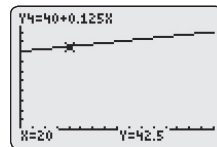
d) Answers may vary. Sample answer: No. Since the Blue party starts off with 40% and the Red party with 20%, there is a good possibility that there are more than two parties. Knowing the percent of undecided voters would help.

6. a)  $R(t) = 20 + 0.375t$ ; increasing at a constant rate



b) 20% c) increasing by 0.375%/day d) Answers may vary. Sample answer: If the election is held before 22.9 days, the Blue party will win. If the election is held after 22.9 days, then the Red party will win. 22.9 days is approximately when the two functions intersect.

7. a)  $V(t) = 40 + 0.125t$ ; popularity of the V party; increasing at a constant rate



b) Answers may vary. Sample answer: The V party will win if the election is held before 80 days. At 80 days, both the Red party and the V party have 50% of the vote, while the Blue party has 0%. c) Answers may vary. Sample answer: If there were a fourth party, it would have 0% of the vote ( $R(t) + B(t) + V(t) = 100$  for  $0 \leq t \leq 80$ ).

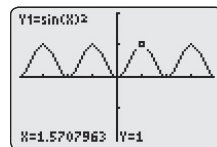
8. No

9. a)  $y = \pm x^{\frac{1}{3}}$  b)  $y = x$  c)  $y = x$  d) same

e) 3, 5, -1,  $f(f^{-1}(x)) = x$  for all values of  $x$ .

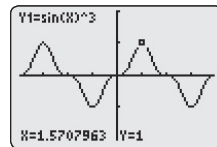
10. Yes

11. a)  $y = \sin^2 x$



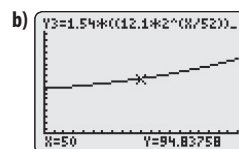
c) Yes d)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, 0 \leq y \leq 1\}$

12. a)  $y = \sin^3 x$ ; periodic;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, -1 \leq y \leq 1\}$



b) Answers may vary. Sample answer: The functions are both periodic and have a maximum value of 1. The functions differ in their minimum values (0 versus  $-1$ ) and period ( $\pi$  versus  $2\pi$ ). One function is even, while the other is odd.

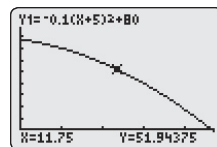
13. a)  $C(P(t)) = 18.634 \times 2^{\frac{t}{52}} + 58.55$



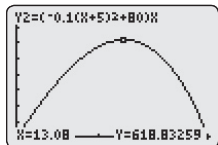
c) 60 years

14.  $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

15. a) parabolic, decreasing



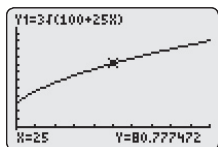
b)  $\{p \in \mathbb{R}, 5 \leq p \leq 23.28\}$  c) No. d)  $R(p) = [-0.1(p + 5)^2 + 80]p$



15. e) \$13.08, \$618.83

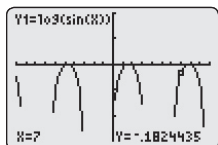
16. a)  $W(N(t)) = 3\sqrt{100 + 25t}$

b)  $\{t \in \mathbb{R}, t \geq 0\}, \{W \in \mathbb{R}, W \geq 30\}$



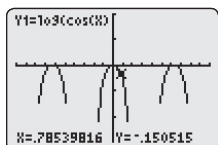
18. a)  $\{x \in \mathbb{R}, x > 0\}$

b), c)  $\{x \in \mathbb{R}, 2n\pi < x < (2n + 1)\pi, n \in \mathbb{Z}\}, \{y \in \mathbb{R}, y < 0\}$



d), e)  $\left\{x \in \mathbb{R}, \frac{(4n - 5)\pi}{2} < x < \frac{(4n - 3)\pi}{2}, n \in \mathbb{Z}\right\},$

$\{y \in \mathbb{R}, y < 0\}$



19. a)  $\{x \in \mathbb{R}, x \neq 0\}, \{y \in \mathbb{R}, y > -9\}$

b)  $\{x \in \mathbb{R}, x \neq -3, x \neq 3\}, \left\{y \in \mathbb{R}, y \leq -\frac{1}{9}, y > 0\right\}$

20.  $d(t) = \sqrt{41t}$

21.  $f^{-1}(x) = \frac{b - dx}{cx - a}$

23. 0, 1,  $\frac{-1 + \sqrt{5}}{2}$

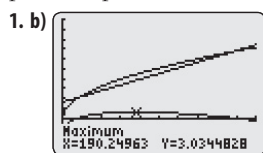
24.  $\frac{7}{25}$

25. 10

### 8.4 Inequalities of Combined Functions, pages 450–460

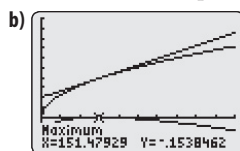
1. a) i) The minimum number of homes will decrease and the maximum number of homes will increase. From the intersection points, the approximate number of homes becomes  $44 \leq n \leq 442$ .

ii) The maximum potential profit will increase. From the maximum of the difference function, the maximum potential profit becomes approximately \$3.0 million.

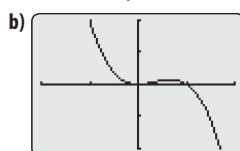


2. a) i) There will be no minimum or maximum number of homes, since the cost is higher than the revenue.

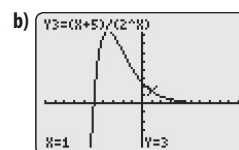
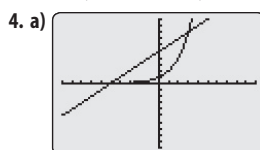
ii) There will be no profit, but a minimum loss of \$153 846.



3. a) i)  $x < 0, 0 < x < 1$  ii)  $x > 1$



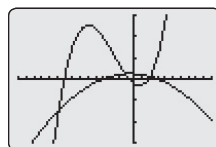
c)  $x < 0, 0 < x < 1$ , same



c) i) When  $y = \frac{u(x)}{v(x)} > 1$ ,  $u(x) > v(x)$  for approximately  $-5 < x < 3$ .

ii) When  $y = \frac{u(x)}{v(x)} > 1$ ,  $u(x) < v(x)$  for approximately  $x < -5, x > 3$ .

5. a) Window variables:  $x \in [-15, 10], y \in [-150, 150], Y_{\text{sc1}} 50$



b) i) approximately  $(-9.77, -1.23)$  or  $(2, \infty)$

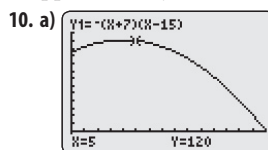
ii) approximately  $(-\infty, -9.77)$  or  $(-1.23, 2)$

7. a) Subtract  $g(x)$  from  $f(x)$ . b)  $y = -x^2 + 5x - 4$

c) Yes. When the graph is below the  $x$ -axis,  $f(x) > g(x)$ . When the graph is above the  $x$ -axis,  $g(x) > f(x)$ .

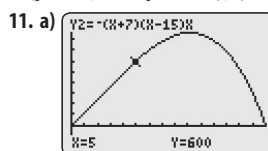
8.  $(0, \infty)$

9. approximately  $(-\infty, -0.77)$  or  $(2, 4)$



b)  $5 \leq p < 15$

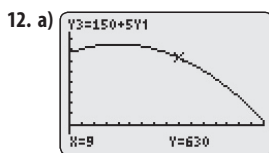
c)  $\{p \in \mathbb{R}, 5 \leq p < 15\}, \{N \in \mathbb{R}, 0 < N \leq 120\}$



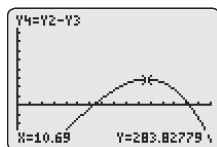
b)  $5 \leq p < 15$ ; yes

c) The  $N(p)$  maximum is at  $(4, 121)$ , while the  $R(p)$  maximum is at approximately  $(9.16, 864.47)$ .

The price affects where the maximum is. d) \$9.16



b) profit function

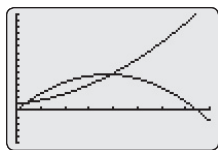


c)  $6.30 < p < 14.24$ ; profit d) No. e) \$10.69, \$283.83

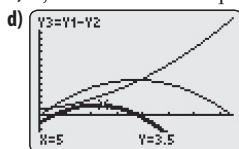
13.  $0 \leq t < 0.22$ ,  $0.74 < t < 1.27$ , and  $1.79 < t \leq 2$

14. Answers may vary. Sample answer:  $f(x) = 5$  and  $g(x) = -1$

15. a) Window variables:  $x \in [0, 16]$ , Xscl 2,  $y \in [-10, 30]$ , Yscl 2



b) 2; where revenue equals cost c)  $0.853 < n < 7.813$ ; profit



e) i) approximately 4333 ii) approximately \$0.84/unit

iii) approximately \$3633 f) Answers may vary.

16. a)  $C(n) = 280 + 8n$  b)  $R(n) = (45 - n)n$  c)  $R(n) > C(n)$

d)  $11 \leq n \leq 26$  e) 18 birdhouses at a profit of \$61

18. Answers may vary. Sample answer:

$f(x) = \sin x$  and  $g(x) = 1$

19. Answers may vary. Sample answers:

a)  $f(x) = 2x$  and  $g(x) = 0.5x^2$  b)  $f(x) = 4x$  and  $g(x) = x^2$

20. a) Answers may vary. Sample answer:

$f(x) = -(x - 3)(x + 3)$  and  $g(x) = x + 3$  b) Yes.

21. Answers may vary. Sample answer:

$f(x) = (x + 3)(x - 4) + 1$  and  $g(x) = 1$

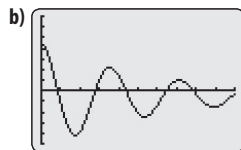
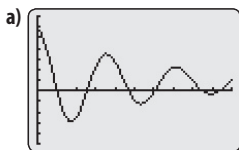
22. 8

23. 3 or -3

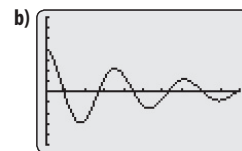
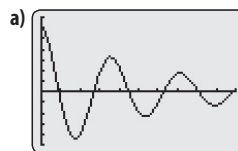
24. 16

### 8.5 Making Connections: Modelling With Combined Functions, pages 461–471

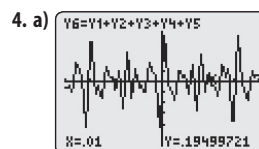
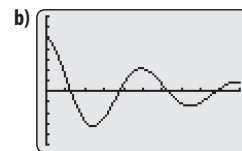
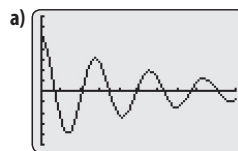
1. Window variables:  $x \in [0, 50]$ , Xscl 5,  $y \in [-100, 140]$ , Yscl 20



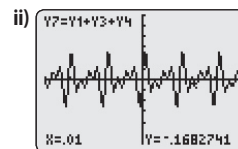
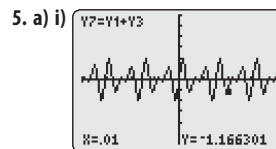
2. Window variables:  $x \in [0, 50]$ , Xscl 5,  $y \in [-100, 140]$ , Yscl 20



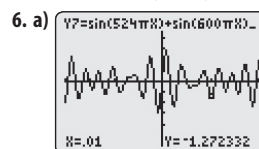
3. Window variables:  $x \in [0, 50]$ , Xscl 5,  $y \in [-100, 140]$ , Yscl 20



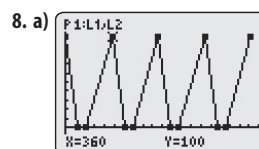
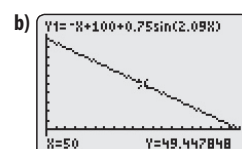
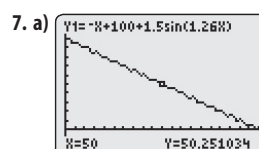
b) Answers may vary.



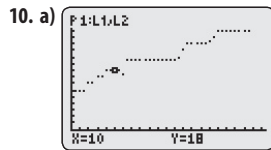
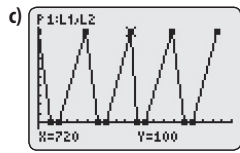
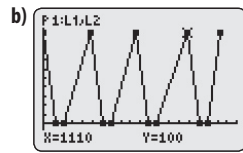
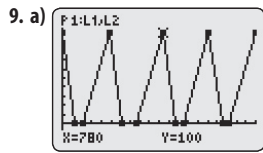
b) Answers may vary.



b) Answers may vary. Sample answer: This is not a pattern like the others.



b) Answers may vary. Sample answer:  $0 < x < 100$ , skier going down the hill;  $100 < x < 160$ , skier in line;  $160 < x < 360$ , skier on lift c) Answers may vary.

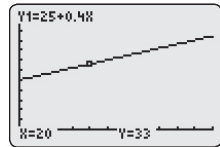


b) Answers may vary. Sample answer: Using regression on the graphing calculator, a curve of best fit that is quadratic has a greater  $R^2$  value.

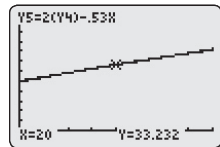
c)  $N(t) \doteq -0.003x^2 + 0.621x + 11.003$

d)  $N(t) \doteq -0.001x^2 + 0.505x + 12.216$ ; yes, it gives a better approximation for each value.

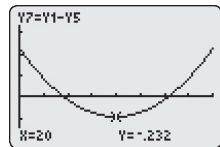
11. a) increasing, producing more high-calibre players



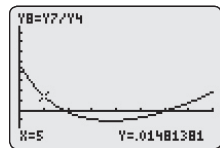
b) almost equal



c) Answers may vary. Sample answer: It has stayed relatively constant over the years. d) confirm e) how many more draftees than retirees there are



f) decreasing and then increasing; surplus, not enough, surplus  
12. a) number of extra players per team over time



b) Years when there are no extra players; answers may vary.

c) Answers may vary. Sample answers:  $P(t)$  increases;  $y = \frac{P(t)}{N(t)}$  increases so that it no longer crosses the  $t$ -axis;

surplus of players to draft

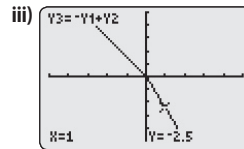
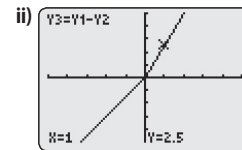
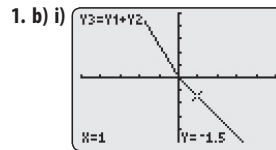
13. Answers may vary.

14. Answers may vary. Sample answer:

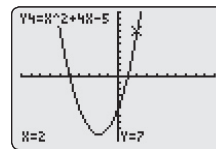
$$y = 120(0.5^{0.1x}) + 2 \sin x$$

15. Answers may vary.

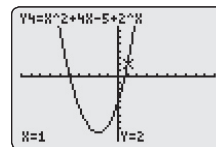
### Chapter 8 Review, pages 472–473



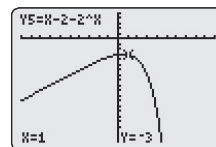
2. a)  $y = x^2 + 4x - 5$ ;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -9\}$



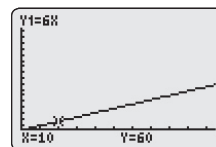
b)  $y = x^2 + 4x - 5 + 2^x$ ;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -8.76\}$



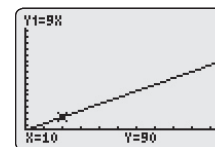
c)  $y = x - 2 - 2^x$ ;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq -2.9\}$



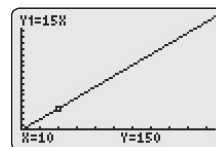
4. a)  $W = 6x$



b)  $T = 9x$

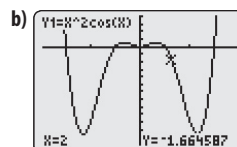


c)  $E = 15x$

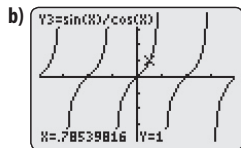
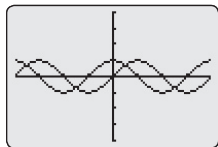


d) \$780

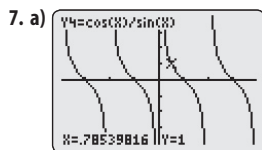
5. a) line symmetry about the  $y$ -axis; both  $u(x)$  and  $v(x)$  are even functions



6. a) Window variables:  $x \in \left[-\frac{47\pi}{24}, \frac{47\pi}{24}\right]$ , Xscl  $\frac{\pi}{2}$ ,  $y \in [-4, 4]$



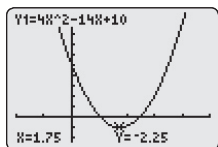
c)  $\left\{x \in \mathbb{R}, x \neq \frac{(2n-1)\pi}{2}, n \in \mathbb{Z}\right\}$ ,  $\{y \in \mathbb{R}\}$  d)  $y = \tan x$



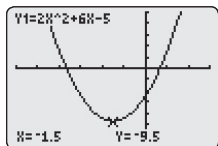
b)  $\{x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}\}$ ,  $\{y \in \mathbb{R}\}$

c) Answers may vary. Sample answer: reflection in the  $y$ -axis and shifted right  $\frac{\pi}{2}$  units

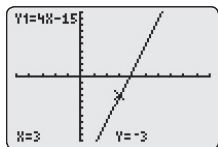
8. a)  $y = 4x^2 - 14x + 10$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq -2.25\}$



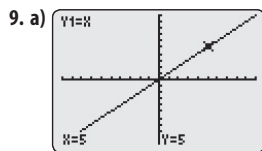
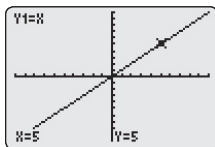
b)  $y = 2x^2 + 6x - 5$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq -9.5\}$



c)  $y = 4x - 15$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$



d)  $y = x$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$



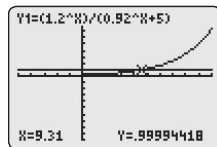
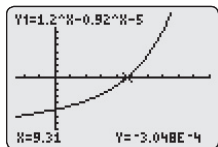
b) Answers may vary. Sample answer:

Given  $f(x) = 2x - 3$ , then  $f^{-1}(x) = \frac{x+3}{2}$  and  $f(f^{-1}(x)) = x$ .

Given  $f(x) = x^2$ , then  $f^{-1}(x) = \pm\sqrt{x}$  and  $f(f^{-1}(x)) = x$ .

10. a) i) approximately  $x > 9.31$  ii)  $x < 9.31$

b) i), ii)



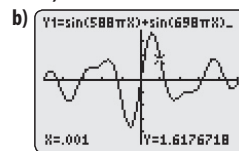
11. Answers may vary.

12. a) i) all values except at point A, where  $C = R$

ii) no values b) not profitable c) Answers may vary.

Sample answer: The business owner should reduce costs.

13. a) 588 Hz



14. Answers may vary.

### Chapter 8 Practice Test, page 474–475

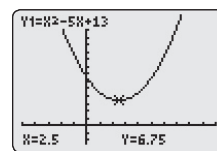
1. A

2. D

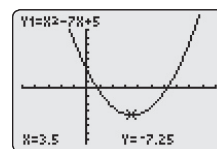
3. D

4. A

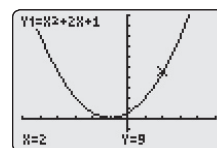
5. a)  $y = x^2 - 5x + 13$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 6.75\}$



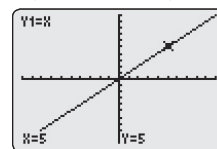
b)  $y = x^2 - 7x + 5$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq -7.25\}$



c)  $y = x^2 + 2x + 1$ ;  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$



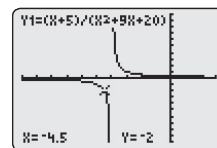
d)  $y = x$ ;  $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$



7. Answers may vary.

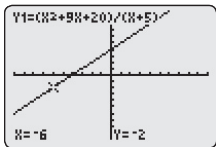
8. a)  $y = \frac{1}{x+4}$ ,  $x \neq -5$ ,  $x \neq -4$ ;  $\{x \in \mathbb{R}, x \neq -5, x \neq -4\}$ ,

$\{y \in \mathbb{R}, y \neq 0, y \neq -1\}$



hole:  $(-5, -1)$ , asymptotes:  $x = -4$  and  $y = 0$

b)  $y = x + 4, x - 5; \{x \in \mathbb{R}, x \neq -5\}, \{y \in \mathbb{R}, y \neq -1\}$

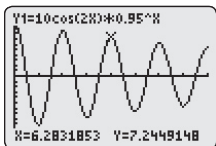


hole:  $(-5, -1)$

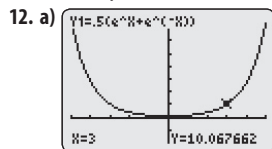
9. a)  $V(C) = \frac{C^3}{6\pi^2}$  b)  $V(SA) = \frac{SA}{3} \sqrt{\frac{SA}{4\pi}}$

10. odd

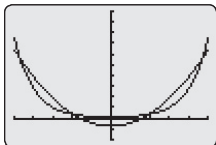
11. a) damped harmonic oscillation;  $\{t \in \mathbb{R}, t \geq 0\}, \{x \in \mathbb{R}, -9.23 \leq x \leq 10\}$



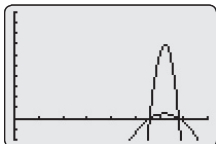
b) i)  $10 \cos(2t)$  ii)  $0.95^t$  c)  $10 \text{ cm}$  d) at  $x(t) = 0$ ; when the pendulum bob crosses the rest position the first time  
e) at crests and troughs; when the pendulum bob changes direction f)  $12.7 \text{ s}$



b) Answers may vary. Sample answer:  $y = 2.855x^2 - 7.217$   
c)  $(-4.73, 56.67), (-1.94, 3.56), (1.94, 3.56), (4.73, 56.67)$   
Window variables:  $x \in [-5, 5], y \in [-20, 100], \text{Yscl } 10$

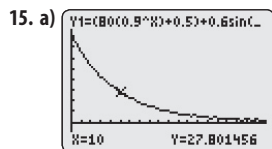


13. a) Window variables:  $x \in [0, 40], \text{Xscl } 5, y \in [-2, 10]$



b)  $31^\circ\text{C}, 45$  c)  $31^\circ\text{C}$ , yes

14. a) Answers may vary. Sample answer:  $f(x) = (x - 1)(x + 1)$  and  $g(x) = 0.5(x - 1)(x + 1)$



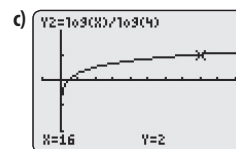
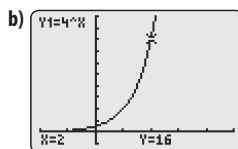
b) Answers may vary. Sample answer: The skier encounters moguls at  $10 \text{ s}$ , where the bumps start on the graph.

c)  $\{t \in \mathbb{R}, 0 \leq t \leq 64.3\}, \{b \in \mathbb{R}, 0 \leq b \leq 80\}$

### Chapters 6 to 8 Review, pages 476–477

1. a)

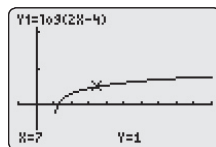
$x$	$y$
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
$\frac{1}{2}$	2
1	4
2	16
3	64



2. a) 2 b) 7 c) -3 d) 3 e) 4 f) -5

3. a) 5.9 units b) 0.9 units

4. a)  $\{x \in \mathbb{R}, x > 2\}$ ,  $x$ -intercept 2.5,  $y$ -intercept none, asymptote  $x = 2$



b) reflection in the  $x$ -axis

5. a) 10 b) 3

6. a) 1.95 b) 4.17

7. a) \$28 000, when  $t = 0$  b) 2.4 years

8. a) 99 decibels b)  $10^{-10} \text{ W/m}^2$

9. a)  $\frac{4}{7}$  b) 1 c)  $x = 17$

10. a) 1.89 b) 2.62 c) -4.15 d) -2.58 e) 5.06 f) 0.33

11. a) 194 h b) 2.8 h

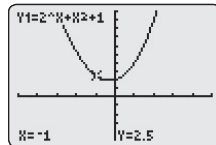
12. a)  $\log(x + 1), x > -1, x \neq 1$  b)  $\log(x^{\frac{3}{2}}), x > 0$   
c)  $2 \log(x + y), x > -y, x \neq 0$

13. a)  $-\frac{4}{3}$  b) 1, extraneous root  $-\frac{5}{3}$  c) 7, extraneous root 1

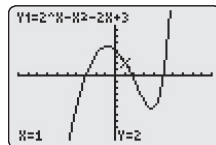
14. Different; the domain of the graph of  $g(x)$  is  $\{x \in \mathbb{R}, x > 0\}$ , while the domain of  $f(x)$  is  $\{x \in \mathbb{R}\}$ .

15. a)  $P(t) = 38\,000(1.12)^t$  b) 6.1 years c) 8.5 years

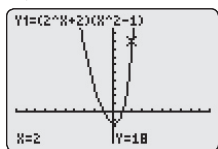
16. a)  $y = 2^x + x^2 + 1; \{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 1.9\}$



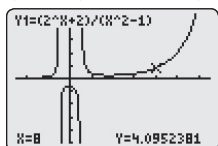
b)  $y = 2^x - x^2 - 2x + 3; \{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$



c)  $y = (2^x + 2)(x^2 - 1)$ ;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -3.04\}$

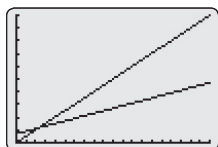


d)  $y = \frac{2^x + 2}{x^2 - 1}$ ;  $\{x \in \mathbb{R}, x \neq -1, x \neq 1\}$ ,  
 $\{y \in \mathbb{R}, y \leq -2.96, y > 0\}$



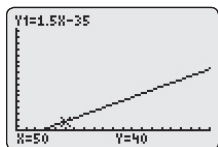
17. a) i)  $C(n) = 35 + n, 0 \leq n \leq 200$

ii)  $R(n) = 2.5n, 0 \leq n \leq 200$  b) Window variables:  
 $x \in [0, 200], Xscl 10, y \in [0, 500], Yscl 50$



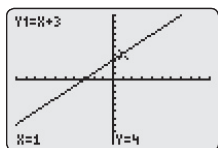
c) (23.33, 58.33); Kathy makes a profit if she sells 24 or more cups of apple cider. Kathy loses money if she sells 23 or fewer cups of apple cider.

d)  $P(n) = 1.5n - 35$

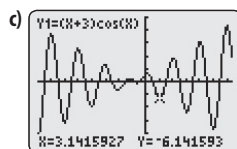
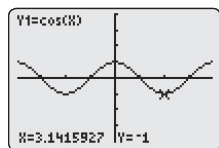


e) \$265

18. a) linear; neither



b) periodic; even

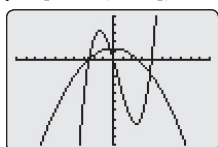


e)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

19. a)  $y = \frac{\sqrt{1-9x^2}}{x}$ ;  $\left\{x \in \mathbb{R}, -\frac{1}{3} \leq x < 0, 0 < x \leq \frac{1}{3}\right\}$ ,

$\{y \in \mathbb{R}\}$  b)  $y = \frac{1}{x-9}$ ;  $\{x \in \mathbb{R}, x \neq 9\}, \{y \in \mathbb{R}, y \neq 0\}$

20. a) Window variables:  $x \in [-20, 20], Xscl 2,$   
 $y \in [-200, 100], Yscl 20$



b)  $[-5, -0.65]$  or  $(7.65, \infty)$

## Course Review, pages 479–483

1. a) An even function is symmetric with respect to the  $y$ -axis. An odd function is symmetric with respect to the origin.

b) Substitute  $-x$  for  $x$  in  $f(x)$ . If  $f(-x) = f(x)$ , the function is even. If  $f(-x) = -f(x)$ , the function is odd.

2. Answers may vary. Sample answer: A polynomial function has the form  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ .

For a polynomial function of degree  $n$ , where  $n$  is a positive integer, the  $n$ th differences are equal (or constant).

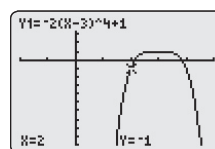
3.  $f(x)$  extends from quadrant 3 to quadrant 4; even exponent, negative coefficient

$g(x)$  extends from quadrant 2 to quadrant 1; even exponent, positive coefficient

$h(x)$  extends from quadrant 3 to quadrant 1; even exponent, positive coefficient

4. 4

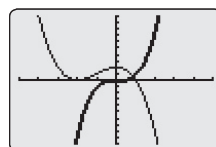
5.  $y = -2(x-3)^4 + 1$



6.  $f(x)$ :  $x$ -intercept 0,  $y$ -intercept 0,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

$g(x)$ :  $x$ -intercepts  $-2$  and  $1$ ,  $y$ -intercept  $2$ ,  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$

Window variables:  $x \in [-20, 20], Xscl 2, y \in [-200, 100], Yscl 20$



7. a) 138 b) 18; 342 c) The graph is increasing for  $1 < x < 3$ .

8. a)  $y = (x+3)(x+1)(x-2)(x-5)$  b)  $y = -(x+5)(x-1)^2$

9. For  $x < 0$ , the slope is positive and decreasing. For  $x > 0$ , the slope is negative and decreasing.

10. a)  $\frac{4x^3 + 6x^2 - 5x + 2}{2x - 1} = 2x^2 + 4x + 4 + \frac{6}{2x - 1}$ ,  
 $x \neq \frac{1}{2}$

b)  $\frac{2x^3 - 4x + 8}{x - 2} = 2x^2 + 4x + 4 + \frac{16}{x - 2}$ ,  $x \neq 2$

c)  $\frac{x^3 - 3x^2 + 5x - 4}{x + 2} = x^2 - 5x + 15 + \frac{-34}{x + 2}$ ,  $x \neq -2$

d)  $\frac{5x^4 - 3x^3 + 2x^2 + 4x - 6}{x + 1} = 5x^3 - 8x^2 + 10x - 6$ ,  $x \neq -1$

11. a)  $(x-1)(x+2)(x+3)$  b)  $(x-3)(x+1)(2x+5)$

c)  $(x-2)(x^2-5x+1)$  d)  $(x^2+x+1)(x^2-x+1)$

12. a) 345 b)  $\frac{44}{27}$

13. a) No. b) No.

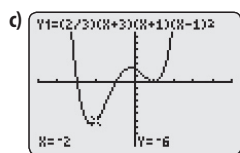
14. a)  $-3, 3$  b)  $-2, -1, 4$  c)  $-\frac{3}{2}$  d)  $-\frac{3}{2}, -\frac{1}{2}, 0, \frac{2}{3}$

15. a) Answers may vary. Sample answer:

$y = k(x+3)(x+1)(x-1)^2$ ;  $y = 2(x+3)(x+1)(x-1)^2$ ,  
 $y = -(x+3)(x+1)(x-1)^2$

b)  $y = \frac{2}{3}(x+3)(x+1)(x-1)^2$





d)  $x < -3, -1 < x < 1, x > 1$

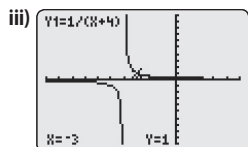
16. a)  $x < -3$  or  $x > 4$  b)  $-2 < x < \frac{3}{2}$

c)  $x \leq -2$  or  $-1 \leq x \leq 5$

17. a)  $x = 2, y = 0$  b)  $x = -3, y = 1$

c)  $x = -3, x = 3, y = 0$  d)  $y = 0$

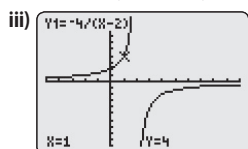
18. a) i)  $x = -4, y = 0$  ii) y-intercept  $\frac{1}{4}$



iv) decreasing for  $x < -4$  and  $x > -4$

v)  $\{x \in \mathbb{R}, x \neq -4\}, \{y \in \mathbb{R}, y \neq 0\}$

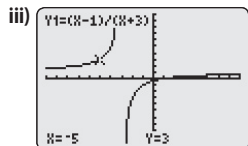
b) i)  $x = 2, y = 0$  ii) y-intercept 2



iv) increasing for  $x < 2$  and  $x > 2$

v)  $\{x \in \mathbb{R}, x \neq 2\}, \{y \in \mathbb{R}, y \neq 0\}$

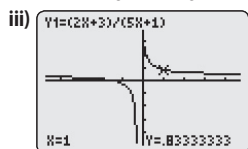
c) i)  $x = -3, y = 1$  ii) y-intercept  $-\frac{1}{3}$ , x-intercept 1



iv) increasing for  $x < -3$  and  $x > -3$

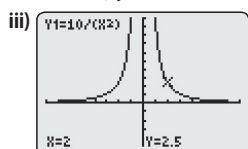
v)  $\{x \in \mathbb{R}, x \neq -3\}, \{y \in \mathbb{R}, y \neq 1\}$

d) i)  $x = -\frac{1}{5}, y = \frac{2}{5}$  ii) y-intercept 3, x-intercept  $-\frac{3}{2}$



iv) decreasing for  $x < -\frac{1}{5}$  and  $x > -\frac{1}{5}$  v)  $\left\{x \in \mathbb{R}, x \neq -\frac{1}{5}\right\}, \left\{y \in \mathbb{R}, y \neq \frac{2}{5}\right\}$

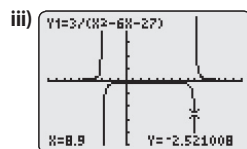
e) i)  $x = 0, y = 0$  ii) no intercepts



iv) increasing for  $x < 0$ , decreasing for  $x > 0$

v)  $\{x \in \mathbb{R}, x \neq 0\}, \{y \in \mathbb{R}, y > 0\}$

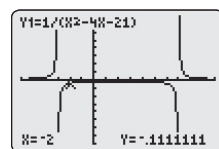
f) i)  $x = -3, x = 9, y = 0$  ii) y-intercept  $-\frac{1}{9}$



iv) increasing for  $x < -3$  and  $-3 < x < 3$ , decreasing for  $3 < x < 9$  and  $x > 9$

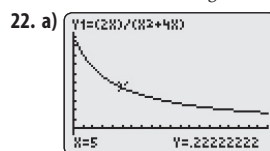
v)  $\{x \in \mathbb{R}, x \neq -3, x \neq 9\}, \left\{y \in \mathbb{R}, y \leq -\frac{1}{12}, y > 0\right\}$

19. positive increasing slope for  $x < -3$ , positive decreasing slope for  $-3 < x < 2$ , negative decreasing slope for  $2 < x < 7$ , negative increasing slope for  $x > 7$



20. a)  $\frac{17}{4}$  b) 7 c) -2

21. a)  $x < 4$  or  $x > \frac{23}{5}$  b)  $x < -4$  or  $-1 < x \leq 3$  or  $x \geq 5$



b)  $R(t) = 0$ ; The chemical will not completely dissolve.

c)  $\{t \in \mathbb{R}, 0 \leq t < 36\}$

23. a)  $\frac{3\pi}{4}$  b)  $-\frac{\pi}{3}$

24. a)  $30^\circ$  b)  $202.5^\circ$

25.  $\frac{15\pi + 72}{4}$

26. a)  $-\frac{\sqrt{3}}{2}$  b) -1 c)  $\sqrt{3}$  d) -1

27. a)  $\frac{1 + \sqrt{3}}{2\sqrt{2}}$  b)  $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

29.  $\frac{\sqrt{11} + 10\sqrt{6}}{30}$

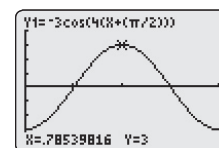
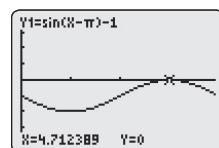
30.  $\frac{\pi}{8}$

31. a) period  $\pi$ , amplitude 3, phase shift  $\frac{\pi}{2}$  rad to the right, vertical translation 4 units upward

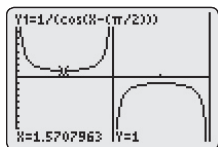
b)  $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, 1 \leq y \leq 7\}$

32. a) x-intercept  $\frac{3\pi}{2}$

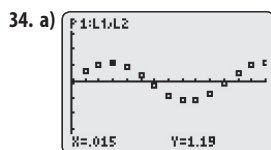
b) x-intercepts  $\frac{\pi}{8}, \frac{3\pi}{8}$



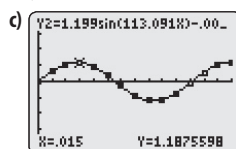
c) asymptotes  $x = 0$ ,  $x = \pi$ ,  $x = 2\pi$



33. a)  $\frac{4\pi}{3}$ ,  $\frac{5\pi}{3}$  b)  $\frac{\pi}{6}$ ,  $\frac{\pi}{2}$ ,  $\frac{5\pi}{6}$ ,  $\frac{3\pi}{2}$  c)  $\frac{\pi}{6}$ ,  $\frac{5\pi}{6}$ ,  $\frac{3\pi}{2}$



b)  $y = 1.199 \sin(113.091x) - 0.002$



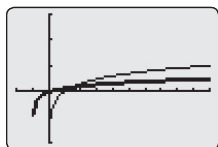
d) 135.6 cm/s

35. a)  $\log_7 49 = 2$  b)  $\log_a c = b$  c)  $\log_8 512 = 3$  d)  $\log_{11} y = x$

36. a)  $f(x)$ :  $x$ -intercept 1, asymptote  $x = 0$

$g(x)$ :  $x$ -intercept 0,  $y$ -intercept 0, asymptote  $x = -1$

Window variables:  $x \in [-2, 10]$ ,  $y \in [-2, 3]$



b)  $f(x)$ :  $\{x \in \mathbb{R}, x > 0\}$ ,  $\{y \in \mathbb{R}\}$ ;  $g(x)$ :  $\{x \in \mathbb{R}, x > -1\}$ ,  $\{y \in \mathbb{R}\}$

37. a)  $3^8 = 6561$  b)  $a^b = 75$  c)  $7^4 = 2401$  d)  $a^b = 19$

38. a) 8 b) 1 c)  $\frac{1}{2}$  d) 5 e) 1 f) -1

39. a) 81 b) 5 c) 16 807 d) 3 e) -7 f) -3

40. 62 min

41. a) alkaline,  $1.585 \times 10^{-8}$  mol/L b) 3.1

42. a)  $x \doteq 1.29$  b)  $x \doteq 0.79$  or  $x \doteq 1.16$

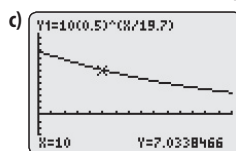
43. a) 3 b)  $\frac{3}{2}$  c) -2 d)  $\frac{1}{3}$

44. a) 3.7004 b) 0.9212 c) 2.6801 d) 0.0283

45. a) 3 b) -2

46.  $x \doteq 1.6180$

47. a)  $h \doteq 19.7$  min b) approximately 7.03 mg

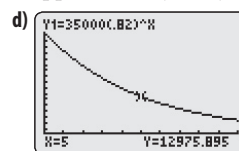


d) Answers may vary. Sample answer: The graph would decrease faster because the sample would be decreasing at a faster rate.

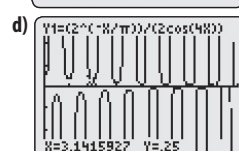
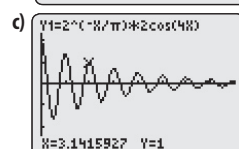
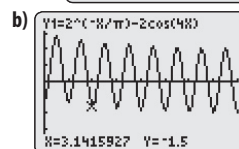
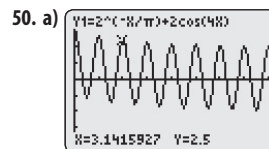
48. a)  $d \doteq 4.04$  years b) approximately 3 229 660

49. a)  $y = 35\,000(0.82)^t$  b) \$12 975.89

c) approximately 3.5 years



e) Answers may vary. Sample answer: The graph would decrease faster.



51. a)  $2x^2 + 15x + 22$  b)  $2x^2 + 3x - 2$  c) -5 d) 117

52. a) 1 b)  $\frac{1}{4}$  c) does not exist d)  $\frac{15}{4}$

53. a)  $f(g(x)) = \sqrt{x+1}$ ,  $\{x \in \mathbb{R}, x \geq -1\}$ ;

$g(f(x)) = \sqrt{x} + 1$ ,  $\{x \in \mathbb{R}, x \geq 0\}$

b)  $f(g(x)) = \sin(x^2)$ ,  $\{x \in \mathbb{R}\}$ ;  $g(f(x)) = \sin^2 x$ ,  $\{x \in \mathbb{R}\}$

c)  $f(g(x)) = |x^2 - 6|$ ,  $\{x \in \mathbb{R}\}$ ;  $g(f(x)) = |x|^2 - 6$ ,  $\{x \in \mathbb{R}\}$

d)  $f(g(x)) = 2^{(3x+3)}$ ,  $\{x \in \mathbb{R}\}$ ;  $g(f(x)) = 3(2^{x+1}) + 2$ ,  $\{x \in \mathbb{R}\}$

e)  $f(g(x)) = 6\sqrt{x-3} + x + 6$ ,  $\{x \in \mathbb{R}, x \geq 3\}$ ;

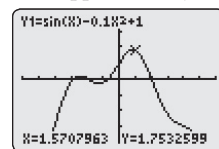
$g(f(x)) = \sqrt{x^2 + 6x + 6}$ ,  $\{x \in \mathbb{R}, x \leq -3 - \sqrt{3}$ ,

$x \geq -3 + \sqrt{3}\}$  f)  $f(g(x)) = (x+1) \log_3 x$ ,  $\{x \in \mathbb{R}\}$ ;

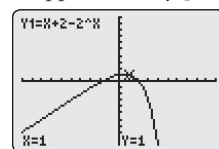
$g(f(x)) = 3^{\log x + 1}$ ,  $\{x \in \mathbb{R}, x > 0\}$

54. a)  $y = -\frac{2}{\sqrt{x}}$  b)  $\{x \in \mathbb{R}, x > 0\}$  c) neither

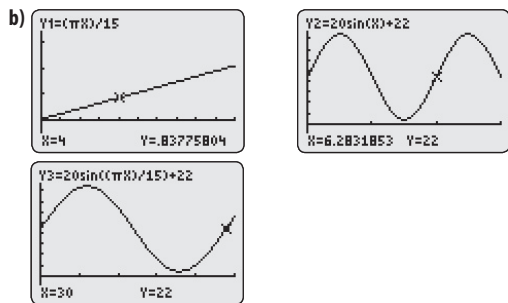
56. a) approximately  $(-\infty, -4.43)$  or  $(-3.11, -1.08)$  or  $(3.15, \infty)$



b) approximately  $[-1.69, 2]$



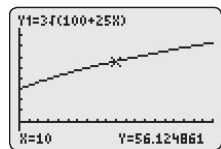
57. a)  $h(t) = 20 \sin\left(\frac{\pi t}{15}\right) + 22$



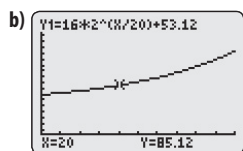
c) The period of  $h(\theta)$  is  $2\pi$  rad. The period of  $h(t)$  is 30 s.

58. a)  $W(t) = 3\sqrt{100 + 25t}$

b)  $(t \in \mathbb{R}, t \geq 0), \{W \in \mathbb{Z}, W \geq 30\}$



59. a)  $C(t) = 16 \times 2^{\frac{t}{20}} + 53.12$



c) approximately 31 years

## PREREQUISITE SKILLS APPENDIX ANSWERS

### Angles From Trigonometric Ratios, page 484

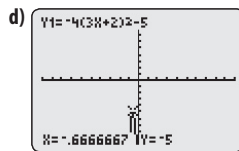
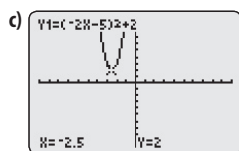
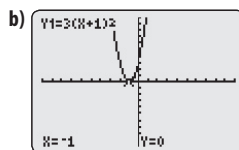
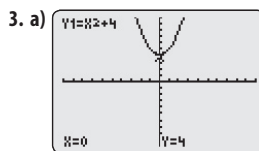
1. a)  $18.8^\circ$  b)  $136.5^\circ$  c)  $70.0^\circ$  d)  $-40.9^\circ$  e)  $75.7^\circ$  f)  $-74.4^\circ$

### Apply the Exponent Laws, pages 484–485

1. a)  $\frac{5}{x^3}$  b)  $\frac{1}{81x^4}$  c)  $7 + \frac{1}{x^6}$  d)  $\frac{1}{25}x^6 - \frac{6}{x} + 2x - x^3$   
 2. a)  $9x^2 + 6x^{\frac{5}{2}} + x^{\frac{1}{2}}$  b)  $6x^5 + 9x^4 - 10x - 15$   
 c)  $4x^6 - 4x^4 + 8x^3 - 8x$  d)  $\sqrt{2x^3 - 4x^2 + 10x - 20}$   
 3. a) 81 b)  $\frac{1}{1024}$  c) 1 d) 6 e) 125  
 4. a)  $20x^9y^7$  b)  $b^3c^3, a, b, c \neq 0$  c)  $m^{-5}n^2, m, n \neq 0$   
 d)  $xy^{-4}, x, y \neq 0$

### Apply Transformations to Functions, pages 485–486

1. a) vertical translation b) vertical stretch c) horizontal compression d) vertical reflection e) horizontal translation f) horizontal reflection g) horizontal translation h) horizontal stretch i) vertical translation j) vertical stretch  
 2. a) vertical stretch by a factor of 3 and horizontal reflection in the y-axis  
 b) vertical translation downward by 3 units and horizontal compression by a factor of  $\frac{1}{2}$   
 c) horizontal translation left by 2 units and vertical reflection in the x-axis  
 d) vertical compression by a factor of  $\frac{1}{3}$ , horizontal compression by a factor of  $\frac{1}{5}$ , and horizontal reflection in the y-axis



### Determine Equations of Quadratic Functions, page 486

1. a)  $f(x) = 2(x - 1)(x - 5)$  b)  $f(x) = -(x + 2)(x - 1)$   
 c)  $f(x) = 1.5(x + 6)(x + 1)$  d)  $f(x) = 0.5(x + 3)(x - 0.5)$

### Determine Intervals From Graphs, page 487

1. a) x-intercepts  $-2$  and  $2$ ; above the x-axis for  $-2 < x < 2$ ; below the x-axis for  $x < -2$  and  $x > 2$   
 b) x-intercepts  $-3, 0$ , and  $3$ ; above the x-axis for  $-3 < x < 0$  and  $x > 3$ ; below the x-axis for  $x < -3$  and  $0 < x < 3$

### Distance Between Two Points, page 487

1. a)  $3\sqrt{2}$  b)  $\sqrt{65}$  c)  $\sqrt{74}$  d)  $3\sqrt{5}$  e)  $3\sqrt{5}$  f)  $\sqrt{82}$

### Domain and Range, page 488

1. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$   
 b)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}$   
 c)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -1\}$   
 d)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 4\}$   
 e)  $\{x \in \mathbb{R} \mid x \geq -5\}, \{y \in \mathbb{R}, y \geq 0\}$   
 f)  $\{x \in \mathbb{R}, x \geq 2\}, \{y \in \mathbb{R}, y \geq 0\}$   
 g)  $\{x \in \mathbb{R}, x \neq -2\}, \{y \in \mathbb{R}, y \neq 0\}$   
 h)  $\{x \in \mathbb{R}, x \neq 1\}, \{y \in \mathbb{R}, y \neq 0\}$

### Equation of a Line, pages 488–489

1. a)  $y = 2x + 1$  b)  $y = -4x + 4$   
 2. a)  $y = 3x - 1$  b)  $y = -x - 3$   
 3. a)  $y = \frac{7}{2}x + \frac{3}{2}$  b)  $y = -x + 8$   
 4. a)  $y = -4x + 4$  b)  $y = \frac{2}{3}x - 2$

### Evaluate Functions, pages 489–490

1. a)  $-7$  b) 35 c) 5 d)  $-\frac{89}{27}$  e) 39.221 f)  $n^3 + 3n^2 - 4n - 7$   
 g)  $-27x^3 + 27x^2 + 12x - 7$  h)  $x^6 + 3x^4 - 4x^2 - 7$   
 2. a)  $-1$  b) 20 c) 2 d)  $x^2 + 4x - 1$   
 3. a) 0 b) 2 c) 1 d)  $\sqrt{x^2 - 3}$   
 4. a) 5 b) 2 c) 10 d) 0  
 5. a) true b) false c) true d) false

**Exact Trigonometric Ratios of Special Angles, pages 490–491**

1.  $\sin 30^\circ = \frac{1}{2}$ ,  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ ,  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ ,  $\csc 30^\circ = 2$ ,  
 $\sec 30^\circ = \frac{2}{\sqrt{3}}$ ,  $\cot 30^\circ = \sqrt{3}$

2.  $\sin 150^\circ = \frac{1}{2}$ ,  $\cos 150^\circ = -\frac{\sqrt{3}}{2}$ ,  $\tan 150^\circ = -\frac{1}{\sqrt{3}}$ ,  
 $\csc 150^\circ = 2$ ,  $\sec 150^\circ = -\frac{2}{\sqrt{3}}$ ,  $\cot 150^\circ = -\sqrt{3}$ ,

$\sin 210^\circ = -\frac{1}{2}$ ,  $\cos 210^\circ = -\frac{\sqrt{3}}{2}$ ,  $\tan 210^\circ = \frac{1}{\sqrt{3}}$ ,  
 $\csc 210^\circ = -2$ ,  $\sec 210^\circ = -\frac{2}{\sqrt{3}}$ ,  $\cot 210^\circ = \sqrt{3}$ ,

$\sin 330^\circ = -\frac{1}{2}$ ,  $\cos 330^\circ = \frac{\sqrt{3}}{2}$ ,  $\tan 330^\circ = -\frac{1}{\sqrt{3}}$ ,  
 $\csc 330^\circ = -2$ ,  $\sec 330^\circ = \frac{2}{\sqrt{3}}$ ,  $\cot 330^\circ = -\sqrt{3}$

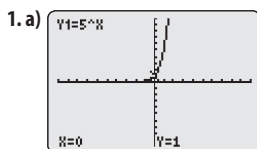
**Factor Quadratic Expressions, pages 491–492**

1. a)  $(x - 2)(x + 2)$  b)  $(y - 10)(y + 10)$  c)  $(2n - 7)(2n + 7)$   
 d)  $(5m - 9)(5m + 9)$  e)  $(1 - 6x)(1 + 6x)$  f)  $(2y - 3x)(2y + 3x)$
2. a)  $(x - 5)(x + 4)$  b)  $(y + 5)(y - 2)$  c)  $(n - 9)(n + 4)$   
 d)  $(m + 3)(m + 6)$  e)  $(x - 6)(x - 5)$  f)  $(c - 6)(c + 4)$   
 g)  $(16 - y)(1 + y)$  h)  $(x + 4y)(x + 8y)$  i)  $(c - 7d)(c + 4d)$
3. a)  $(x - 2)(3x + 4)$  b)  $(2c - 1)(c + 4)$  c)  $(4m - 3)(m - 2)$   
 d)  $(y + 1)(5y + 3)$  e)  $(n + 1)(3n - 2)$  f)  $(6x + 1)(x - 3)$   
 g)  $(3x + 4y)(x - 3y)$  h)  $(5x - 4)(x - 2)$  i)  $(4x + 3)(x + 5)$   
 j)  $(p + q)(2p - q)$

**Finite Differences, pages 492–493**

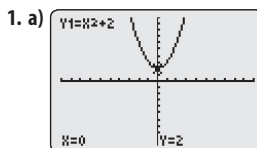
1. a) linear b) quadratic c) linear d) neither e) quadratic f) neither

**Graph an Exponential Function, pages 493–494**

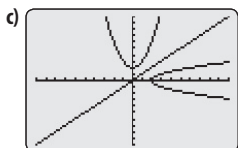


- b)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y > 0\}$ , asymptote  $y = 0$   
 c) i) 1.4 ii) 0.2

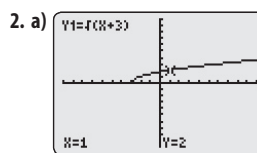
**Graph an Inverse, page 494**



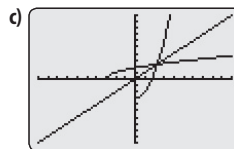
- b)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 2\}$



- d)  $\{x \in \mathbb{R}, x \geq 2\}$ ,  $\{y \in \mathbb{R}\}$   
 e) Since the graph of the inverse does not pass the vertical line test,  $f^{-1}$  is not a function.



- b)  $\{x \in \mathbb{R}, x \geq -3\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$

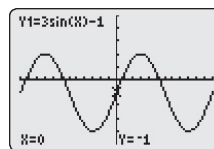


- d)  $\{x \in \mathbb{R}, x \geq 0\}$ ,  $\{y \in \mathbb{R}, y \geq -3\}$

e) Yes. The graph of the inverse passes the vertical line test.

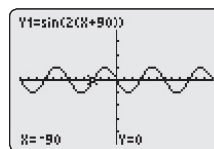
**Graphs and Transformations of Sinusoidal Functions Using Degree Measure, page 495**

1. a) Window variables:  $x \in [-360, 360]$ , Xscl 30,  $y \in [-5, 5]$

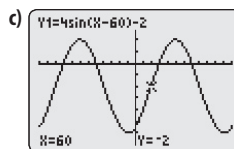


amplitude 3, period  $360^\circ$ , no phase shift, vertical translation 1 unit downward

- b) Window variables:  $x \in [-360, 360]$ , Xscl 30,  $y \in [-5, 5]$

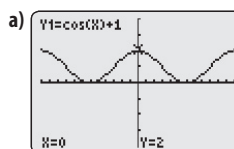


amplitude 1, period  $180^\circ$ , phase shift  $90^\circ$  to the left, no vertical translation

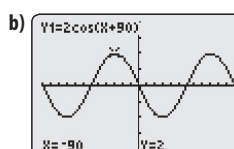


amplitude 4, period  $360^\circ$ , phase shift  $60^\circ$  to the right, vertical translation 2 units downward

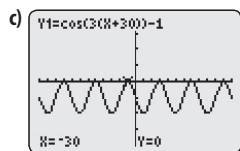
2. Window variables:  $x \in [-360, 360]$ , Xscl 30,  $y \in [-4, 4]$



amplitude 1, period  $360^\circ$ , no phase shift, vertical translation 1 unit upward

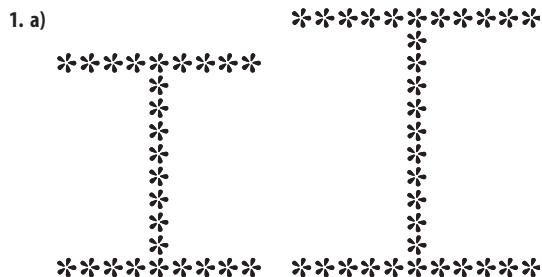


amplitude 2, period  $360^\circ$ , phase shift  $90^\circ$  to the left, no vertical translation



amplitude 1, period  $120^\circ$ , phase shift  $30^\circ$  to the left, vertical translation 1 unit downward

**Identify Linear, Quadratic, and Exponential Growth Models, page 496**

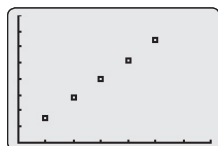


b)

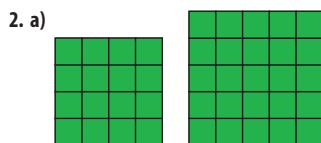
$n$	$t$
1	8
2	14
3	20
4	26
5	32

c) linear

d) Window variables:  $x \in [0, 7]$ ,  $y \in [0, 40]$ , Yscl 5



e)  $t = 6n + 2$

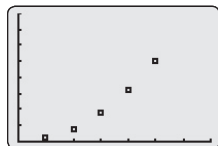


b)

$n$	$t$
1	1
2	4
3	9
4	16
5	25

c) quadratic

d) Window variables:  $x \in [0, 7]$ ,  $y \in [0, 40]$ , Yscl 5



e)  $t = n^2$

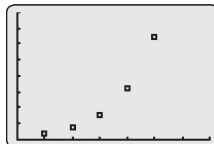
3.

b)

$n$	$t$
1	2
2	4
3	8
4	16
5	32

c) exponential

d) Window variables:  $x \in [0, 7]$ ,  $y \in [0, 40]$ , Yscl 5



e)  $t = 2^n$

**Inverses, pages 496–497**

1. a)  $y = \frac{x+1}{3}$ ; function b)  $y = \frac{x-5}{6}$ ; function

c)  $y = \pm\sqrt{\frac{x-2}{4}}$ ; not a function d)  $y = \pm\sqrt{\frac{x+8}{3}}$ ; not a function e)  $y = \frac{1}{x} + 1$ ; function

**Primary Trigonometric Ratios and the CAST Rule, pages 497–498**

1. a)  $\sin \theta = \frac{8}{17}$ ,  $\cos \theta = \frac{15}{17}$  b)  $\cos \theta = -\frac{12}{13}$ ,  $\tan \theta = -\frac{5}{12}$

c)  $\sin \theta = -\frac{\sqrt{5}}{3}$ ,  $\tan \theta = \frac{\sqrt{5}}{2}$  d)  $\sin \theta = -\frac{8}{\sqrt{89}}$ ,  $\cos \theta = \frac{5}{\sqrt{89}}$

2. a) positive, 0.2867 b) positive, 0.9397 c) negative,  $-0.5736$

d) positive, 0.2867 e) negative,  $-0.4384$  f) negative,  $-0.8192$

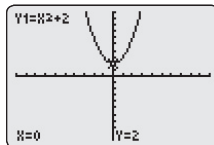
**Product of Two Binomials, page 498**

1. a)  $2a^2 + a - 15$  b)  $3m^2 + 14m + 8$  c)  $2x^2 + 2xy - 12y^2$

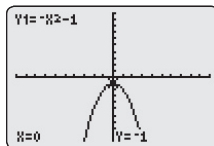
d)  $10m^2 - 11mn + 3n^2$

**Quadratic Functions, pages 498–499**

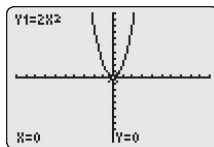
1. a) vertex (0, 2); opens upward; vertical stretch factor 1



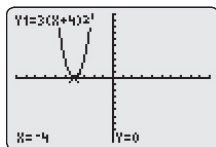
b) vertex (0, -1); opens downward; vertical stretch factor 1



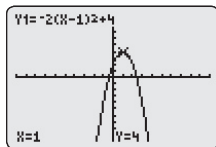
c) vertex (0, 0); opens upward; vertical stretch factor 2



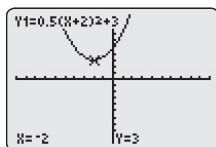
d) vertex  $(-4, 0)$ ; opens upward; vertical stretch factor 3



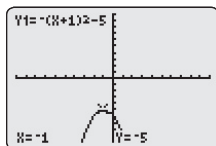
e) vertex  $(1, 4)$ ; opens downward; vertical stretch factor 2



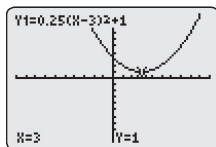
f) vertex  $(-2, 3)$ ; opens upward; vertical stretch factor 0.5



g) vertex  $(-1, -5)$ ; opens downward; vertical stretch factor 1



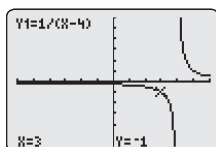
h) vertex  $(3, 1)$ ; opens upward; vertical stretch factor  $\frac{1}{4}$



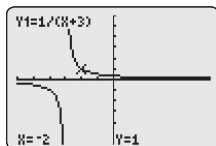
### Reciprocal Functions, page 499

1. Window variables:  $x \in [-6, 6]$ ,  $y \in [-6, 6]$

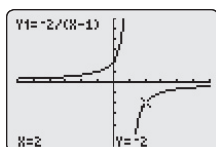
a) vertical asymptote  $x = 4$ , horizontal asymptote  $y = 0$



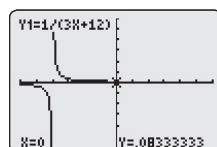
b) vertical asymptote  $x = -3$ , horizontal asymptote  $y = 0$



c) vertical asymptote  $x = 1$ , horizontal asymptote  $y = 0$



d) vertical asymptote  $x = -4$ , horizontal asymptote  $y = 0$



### Reciprocal Trigonometric Ratios, pages 499–501

1. a)  $\csc \theta = \frac{17}{15}$ ,  $\sec \theta = \frac{17}{8}$  b)  $\sec \theta = -\frac{25}{7}$ ,  $\cot \theta = -\frac{7}{24}$   
 c)  $\csc \theta = -\frac{8}{\sqrt{55}}$ ,  $\cot \theta = \frac{3}{\sqrt{55}}$  d)  $\csc \theta = -\frac{\sqrt{13}}{2}$ ,  $\sec \theta = \frac{\sqrt{13}}{3}$   
 2. a) 1.3270 b) -2.4586 c) -7.1853 d) -1.0724 e) -1.7013 f) -1.0038  
 3. a)  $29^\circ$  b)  $105^\circ$  c)  $-80^\circ$  d)  $23^\circ$

### Simplify a Radical Expression, page 501

1. a)  $2\sqrt{3}$  b)  $2\sqrt{5}$  c)  $\frac{3}{4}$  d)  $-\frac{\sqrt{3}}{5}$  e)  $5 + \sqrt{5}$  f)  $3 - \sqrt{6}$

### Simplify Expressions, page 501

1. a)  $5x + 20$  b)  $7y - 30$  c)  $8x - 18$  d)  $4w + 11$   
 e)  $35 - 6y$  f)  $7x^2 - 2x + 4$   
 g)  $4x^2 + 6x - 46$  h)  $5a^2 - 16a + 20$

### Slope, page 502

1. a)  $\frac{1}{2}$  b)  $-\frac{3}{5}$  c)  $\frac{11}{3}$  d)  $\frac{3}{4}$  e)  $-\frac{2}{9}$  f)  $\frac{3}{4}$  e)  $-2$  f)  $-2$

### Slope and y-intercept of a Line, page 502

1. a) slope 4; y-intercept 1 b) slope 1; y-intercept -2  
 c) slope 3; y-intercept 5 d) slope -7; y-intercept 3  
 e) slope 3; y-intercept 0 f) slope 0; y-intercept -8  
 g) slope 5; y-intercept 2 h) slope 4; y-intercept -3  
 i) slope -3.5; y-intercept 2.5 j) slope 1; y-intercept 0  
 k) slope 4; y-intercept 4 l) slope 6; y-intercept 4

### Solve Quadratic Equations, page 503

1. a) -4, -5 b) 1, 2 c) 3, -10 d) -3, -5  
 2. a) -1,  $\frac{2}{3}$  b)  $\frac{5}{2}$  c)  $\pm\frac{3}{5}$  d) 0,  $\frac{4}{9}$   
 3. a) 1,  $\frac{1}{2}$  b)  $\frac{3}{2}, \frac{3}{5}$  c) 1,  $\frac{2}{3}$   
 4. a)  $\frac{-1 \pm \sqrt{11}}{5}$ ; 0.5 or -0.9 b)  $\frac{1 \pm \sqrt{7}}{3}$ ; 1.2 or -0.5  
 c)  $\frac{4 \pm \sqrt{2}}{2}$ ; 2.7 or 1.3 d)  $\frac{-1 \pm \sqrt{15}}{2}$ ; 1.4 or -2.4  
 e)  $\frac{1 \pm \sqrt{11}}{5}$ ; 0.9 or -0.5 f)  $\frac{-5 \pm \sqrt{97}}{12}$ ; 0.4 or -1.2

### Trigonometric Identities, page 504

1. Answers may vary.

### Use Long Division, page 504

1. a) 28 R3 b) 156 R3 c) 146 R17 d) 127 R11